

A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations Made at the Total Eclipse of May 29, 1919

F. W. Dyson, A. S. Eddington and C. Davidson

Phil. Trans. R. Soc. Lond. A 1920 **220**, doi: 10.1098/rsta.1920.0009, published 1 January 1920

References

Article cited in:

<http://rsta.royalsocietypublishing.org/content/220/571-581/291.citation#related-urls>

Email alerting service

Receive free email alerts when new articles cite this article - sign up in the box at the top right-hand corner of the article or click [here](#)

IX. *A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations made at the Total Eclipse of May 29, 1919.*

By Sir F. W. DYSON, F.R.S., Astronomer Royal, Prof. A. S. EDDINGTON, F.R.S., and Mr. C. DAVIDSON.

(Communicated by the Joint Permanent Eclipse Committee.)

Received October 30,—Read November 6, 1919.

[PLATE 1.]

CONTENTS.

	Page
I. Purpose of the Expeditions	291
II. Preparations for the Expeditions	293
III. The Expedition to Sobral	296
IV. The Expedition to Principe	312
V. General Conclusions	330

I. PURPOSE OF THE EXPEDITIONS.

1. THE purpose of the expeditions was to determine what effect, if any, is produced by a gravitational field on the path of a ray of light traversing it. Apart from possible surprises, there appeared to be three alternatives, which it was especially desired to discriminate between—

- (1) The path is uninfluenced by gravitation.
- (2) The energy or mass of light is subject to gravitation in the same way as ordinary matter. If the law of gravitation is strictly the Newtonian law, this leads to an apparent displacement of a star close to the sun's limb amounting to $0''.87$ outwards.
- (3) The course of a ray of light is in accordance with EINSTEIN'S generalised relativity theory. This leads to an apparent displacement of a star at the limb amounting to $1''.75$ outwards.

In either of the last two cases the displacement is inversely proportional to the distance of the star from the sun's centre, the displacement under (3) being just double the displacement under (2).

It may be noted that both (2) and (3) agree in supposing that light is subject to gravitation in precisely the same way as ordinary matter. The difference is that, whereas (2) assumes the Newtonian law, (3) assumes EINSTEIN'S new law of gravitation. The slight

deviation from the Newtonian law, which on EINSTEIN'S theory causes an excess motion of perihelion of Mercury, becomes magnified as the speed increases, until for the limiting velocity of light it doubles the curvature of the path.

2. The displacement (2) was first suggested by Prof. EINSTEIN* in 1911, his argument being based on the Principle of Equivalence, viz., that a gravitational field is indistinguishable from a spurious field of force produced by an acceleration of the axes of reference. But apart from the validity of the general Principle of Equivalence there were reasons for expecting that the electromagnetic energy of a beam of light would be subject to gravitation, especially when it was proved that the energy of radio-activity contained in uranium was subject to gravitation. In 1915, however, EINSTEIN found that the general Principle of Equivalence necessitates a modification of the Newtonian law of gravitation, and that the new law leads to the displacement (3).

3. The only opportunity of observing these possible deflections is afforded by a ray of light from a star passing near the sun. (The maximum deflection by Jupiter is only $0''.017$.) Evidently, the observation must be made during a total eclipse of the sun.

Immediately after EINSTEIN'S first suggestion, the matter was taken up by Dr. E. FREUNDLICH, who attempted to collect information from eclipse plates already taken; but he did not secure sufficient material. At ensuing eclipses plans were made by various observers for testing the effect, but they failed through cloud or other causes. After EINSTEIN'S second suggestion had appeared, the Lick Observatory expedition attempted to observe the effect at the eclipse of 1918. The final results are not yet published. Some account of a preliminary discussion has been given,† but the eclipse was an unfavourable one, and from the information published the probable accidental error is large, so that the accuracy is insufficient to discriminate between the three alternatives.

4. The results of the observations here described appear to point quite definitely to the third alternative, and confirm EINSTEIN'S generalised relativity theory. As is well-known the theory is also confirmed by the motion of the perihelion of Mercury, which exceeds the Newtonian value by $43''$ per century—an amount practically identical with that deduced from EINSTEIN'S theory. On the other hand, his theory predicts a displacement to the red of the Fraunhofer lines on the sun amounting to about 0.008 \AA in the violet. According to Dr. ST. JOHN‡ this displacement is not confirmed. If this disagreement is to be taken as final it necessitates considerable modifications of EINSTEIN'S theory, which it is outside our province to discuss. But, whether or not changes are needed in other parts of the theory, it appears now to be established that EINSTEIN'S law of gravitation gives the true deviations from the Newtonian law both for the relatively slow-moving planet Mercury and for the fast-moving waves of light.

It seems clear that the effect here found must be attributed to the sun's gravitational field and not, for example, to refraction by coronal matter. In order to produce the

* 'Annalen der Physik,' vol. XXXV, p. 898.

† 'Observatory,' vol. XLII, p. 298.

‡ 'Astrophysical Journal,' vol. XLVI, p. 249.

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 293

observed effect by refraction, the sun must be surrounded by material of refractive index $1 + \cdot 00000414/r$, where r is the distance from the centre in terms of the sun's radius. At a height of one radius above the surface the necessary refractive index $1 \cdot 00000212$ corresponds to that of air at $\frac{1}{140}$ atmosphere, hydrogen at $\frac{1}{80}$ atmosphere, or helium at $\frac{1}{20}$ atmospheric pressure. Clearly a density of this order is out of the question.

II. PREPARATIONS FOR THE EXPEDITIONS.

5. In March, 1917,* it was pointed out as the result of an examination of the photographs taken with the Greenwich astrographic telescope at the eclipse of 1905 that this instrument was suitable for the photography of the field of stars surrounding the sun in a total eclipse. Attention was also drawn to the importance of observing the eclipse of May 29, 1919, as this afforded a specially favourable opportunity owing to the unusual number of bright stars in the field, such as would not occur again for many years.

With weather conditions as good as those at Sfax in the 1905 eclipse—and these were by no means perfect—it was anticipated that twelve stars would be shown. Their positions are indicated in the diagram on next page, on which is also marked on the same scale the outline of a 16×16 cm. plate (used with the astrographic telescopes of 3·43 metres focal length) and a 10×8 -inch plate (used with a 4-inch lens of 19 feet focal length).

The following table gives the photographic magnitudes and standard co-ordinates of the stars, and the gravitational displacements in x and y calculated on the assumption of a radial displacement $1'' \cdot 75 \frac{r_0}{r}$, where r is the distance from the sun's centre and r_0 the radius of the sun.

TABLE I.

No.	Names.	Photog. Mag.	Co-ordinates. Unit = 50'.		Gravitational displacement.			
					Sobral.		Principe.	
			x .	y .	x .	y .	x .	y .
		m.			"	"	"	"
1	B.D., 21°, 641	7·0	+0·026	−0·200	−1·31	+0·20	−1·04	+0·09
2	Piazzi, IV, 82	5·8	+1·079	−0·328	+0·85	−0·09	+1·02	−0·16
3	κ^2 Tauri	5·5	+0·348	+0·360	−0·12	+0·87	−0·28	+0·81
4	κ^1 Tauri	4·5	+0·334	+0·472	−0·10	+0·73	−0·21	+0·70
5	Piazzi, IV, 61	6·0	−0·160	−1·107	−0·31	−0·43	−0·31	−0·38
6	ν Tauri	4·5	+0·587	+1·099	+0·04	+0·40	+0·01	+0·41
7	B.D., 20°, 741	7·0	−0·707	−0·864	−0·38	−0·20	−0·35	−0·17
8	B.D., 20°, 740	7·0	−0·727	−1·040	−0·33	−0·22	−0·29	−0·20
9	Piazzi, IV, 53	7·0	−0·483	−1·303	−0·26	−0·30	−0·26	−0·27
10	72 Tauri	5·5	+0·860	+1·321	+0·09	+0·32	+0·07	+0·34
11	66 Tauri	5·5	−1·261	−0·160	−0·32	+0·02	−0·30	+0·01
12	53 Tauri	5·5	−1·311	−0·918	−0·28	−0·10	−0·26	−0·09
13	B.D., 22°, 688	8·0	+0·089	+1·007	−0·17	+0·40	−0·14	+0·39

* 'Monthly Notices, R.A.S.,' LXXVII, p. 445.

It may be noted that No. 1 is lost in the corona on the photographs taken at Sobral. The star, No. 13, of magnitude 8.0, is shown on some of the astrographic plates at Sobral.

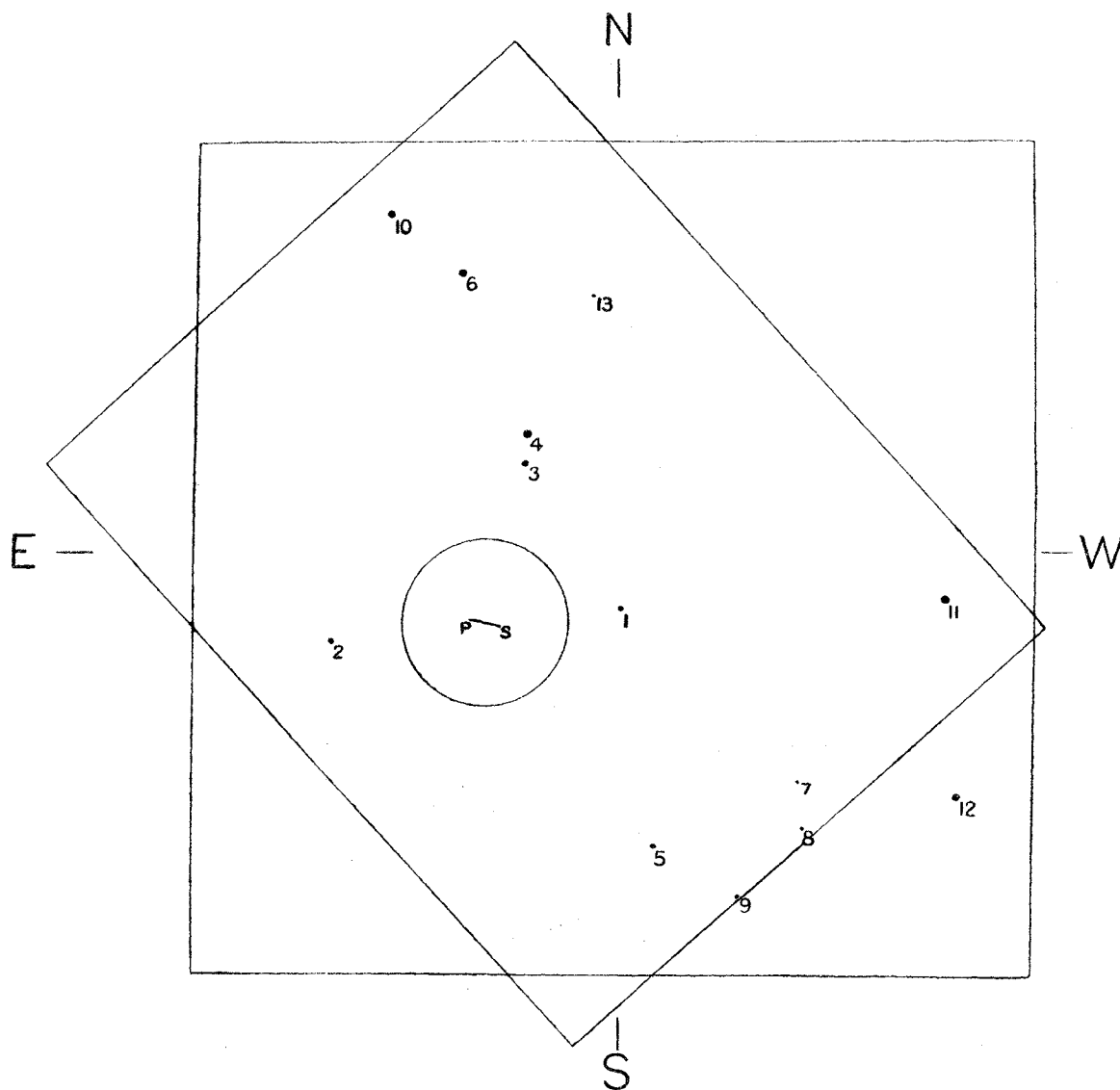


Diagram 1.

6. The track of the eclipse runs from North Brazil across the Atlantic, skirting the African coast near Cape Palmas, passing through the Island of Principe, then across Africa to the western shores of Lake Tanganyika. Enquiry as to the suitable sites and probable weather conditions was kindly made by Mr. HINKS. It appeared that a station in North Brazil, the Island of Principe, and a station on the west of Lake Tanganyika were possible. A station near Cape Palmas did not seem desirable from the meteorological reports though, as the event proved, the eclipse was observed in a cloudless sky

by Prof. BAUER, who was there on an expedition to observe magnetic effects. At the station at Tanganyika it was thought the sun was at too low an altitude for observations of this character, owing to the large displacements which would be caused by refraction.

A circular received from Dr. MORIZE, the director of the Observatory at Rio, stated that Sobral was the most suitable station in North Brazil and gave copious information of the meteorological conditions, mode of access, &c.

7. Acting on this information the Joint Permanent Eclipse Committee at a meeting on November 10, 1917, decided, if possible, to send expeditions to Sobral in North Brazil, and to the island of Principe. Application was made to the Government Grant Committee for £100 for instruments and £1,000 for the expedition, and a sub-committee consisting of Sir F. W. DYSON, Prof. EDDINGTON, Prof. FOWLER and Prof. TURNER was appointed to make arrangements for the expeditions. This sub-committee met in May and June, 1918, and made provisional arrangements for Prof. EDDINGTON and Mr. COTTINGHAM to take the object glass of the Oxford astrographic telescope to Principe, and Mr. DAVIDSON and Father CORTIE to take the object glass of the Greenwich astrographic telescope to Sobral. It was arranged for the clocks and mechanism of the cœlostats to be overhauled by Mr. COTTINGHAM. Preliminary inquiries were also set on foot as to shipping facilities, from which it appeared very doubtful whether the expeditions could be carried through.

Conditions had changed materially in November, 1918, and at a meeting of the sub-committee on November 8, it was arranged to assemble the instruments at Greenwich, and make necessary arrangements with all speed for the observers to leave England by the end of February, 1919. In addition to the astrographic object glasses fed by 16-inch cœlostats, Father CORTIE suggested to the sub-committee the use of the 4-inch telescope of 19-feet focus, which he had used at Hernosand, Sweden, in 1914, in conjunction with an 8-inch cœlostad, the property of the Royal Irish Academy. It was arranged to ask for the loan of these instruments. As Father CORTIE found it impossible to spare the necessary time for the expedition his place was taken by Dr. CROMMELIN of the Royal Observatory.

8. In November, 1918, the only workman available at the Royal Observatory was the mechanic, the carpenter not having been released from military service. In these circumstances Mr. BOWEN, the civil engineer at the Royal Naval College, was consulted. He kindly undertook the construction of frame huts covered with canvas, which could be easily packed and readily put together. These were generally similar to those used in previous expeditions from the Royal Observatory (see 'Monthly Notices,' Vol. LVII., p. 101). He also lent the services of a joiner who worked at the Observatory on the woodwork of the instruments.

It was found possible to obtain steel tubes for the astrographic objectives. These were, for convenience of carriage, made in two sections which could be bolted together. The tubes were provided with flanges at each end, the objective being attached to one of these, and a wooden breech piece to the other. In the breech piece suitable provision

was made for the focussing and squaring on of the plates. The plate holders were of a simple construction, permitting the plate to be pushed into contact with three metal tilting screws on the breech piece thus insuring a constancy of focal plane. Eighteen plate-carriers were obtained for each of the astrographic telescopes, made according to a pattern supplied.

With the 4-inch lens FATHER CORTIE lent the square wooden tube used by him in 1914. This was modified at the breech end to secure greater rigidity and constancy of focus.

It was designed for dark slides carrying 10×8 inch plates, and four of these, carrying eight plates, were lent with the telescope. The desirability of using larger plates was considered, but the time at disposal to make the necessary alterations was insufficient.

The 16-inch coelostats which had been overhauled by Mr. COTTINGHAM were mounted and tested as far as the unfavourable weather conditions of February, 1919, would permit. The 8-inch coelostat was constructed for these latitudes. To make it serviceable near the equator a strong wooden wedge was made on which the coelostat was bolted.

The 8-inch mirror was silvered at the observatory, but owing to lack of facilities for maintaining a uniform temperature approaching 60° F. in the wintry weather of February, the larger mirrors were sent away to be silvered.

Photographic plates, suitably packed in hermetically sealed tin boxes, were obtained from the Ilford and Imperial Companies. The Ilford plates employed were Special Rapid and Empress, and those of the Imperial Company, Special Sensitive, Sovereign and Ordinary.

The instruments were carefully packed and sent to Liverpool a week in advance, with the exception of the objectives. These were packed in cases inside hampers and remained under the personal care of the observers, who embarked on the "Anselm" on March 8.

III. THE EXPEDITION TO SOBRAL.

(*Observers, Dr. A. C. D. CROMMELIN and Mr. C. DAVIDSON.*)

9. Sobral is the second town of the State of Ceara, in the north of Brazil. Its geographical co-ordinates are: longitude 2h. 47m. 25s. west; latitude $3^{\circ} 41' 33''$ south; altitude 230 feet. Its climate is dry and though hot not unhealthy.

The expedition reached Para on the "Anselm" on March 23. There was a choice of proceeding immediately to Sobral or waiting for some weeks. It was considered undesirable to go there before we heard from Dr. MORIZE what arrangements were being made, so we reported our arrival to him by telegram and decided to await his reply. As we had thus some time on our hands we continued the voyage to Manaos in the "Anselm," returning to Para on April 8.

By the courtesy of the Brazilian Government our heavy baggage was passed through the customs without examination and we continued our journey to Sobral, leaving Para on April 24 by the steamer "Fortaleza" and arriving at Camocim on April 29.

Here we were met by Mr. JOHN NICOLAU, who had been instructed to assist us with our baggage through to Sobral. We proceeded from Camocim to Sobral by train on April 30, our baggage following the next day.

We were met at Sobral station by representatives of both the Civil and Ecclesiastical Authorities, headed respectively by Dr. JACOME D'OLIVEIRA, the Prefect, and Mgr. FERREIRA, and conducted to the house which had been placed at our disposal by the kindness of its owner, Col. VICENTE SABOYA, the Deputy for Sobral. We were joined there nine days later by the Washington (Carnegie) Eclipse Commission, consisting of Messrs. DANIEL WISE and ANDREW THOMSON.

We are greatly indebted to Dr. LEOCADIO ARAUJO, of the State Ministry of Agriculture, who had been deputed to interpret for us and to assist us in our preparations. His services were invaluable, and contributed greatly to our success, as also to our well-being during our stay.

10. A convenient site for the eclipse station offered itself just in front of the house; this was the race-course of the Jockey Club, and was provided with a covered grand stand, which we found most convenient for unpacking and storage and in the preparatory work. We laid down a meridian line, after which brick piers were constructed for the cœlostats and for the steel tube of the astrographic telescope. Whilst this was in progress the huts were being erected.

The pier of the small cœlostat was constructed so as to leave a clear space in the middle of one end for the fall of the weight, which was thus below the driving barrel of the clock. By continuing the hole below the foundations of the pier, space was provided for a fall of the weight permitting a run of 25 minutes. In the case of the 16-inch cœlostat, the clock was mounted on the top of a long wooden trunk, nearly 4 feet in length, which was placed on end, and sunk in the earth to a depth of about 2 feet. The weight descended inside the trunk directly from the driving barrel, and had space for a continuous run of over half-an-hour.

The 16-inch cœlostat had free adjustment for all latitudes; but the 8-inch one, constructed for European latitudes, was mounted on a wooden base, inclined at an angle of about 40 degrees, constructed before leaving Greenwich. The clock had to be separated from the cœlostat, mounted on a wooden base and reversed, to adjust to the Southern Hemisphere. It performed very satisfactorily, and no elongation of the star images is shown with 28 seconds' exposure.

To provide for the changing declination of the sun the piers of the astrographic telescope were made with grooves in the top, in which the wooden V-supports of the tube could slide, thus allowing for the change of azimuth.

The tube of the astrographic telescope was circular in section, and could rest in any position in the Vs; for convenience it was adjusted so that the directions of R.A. and declination were parallel to the sides of the plate; this involved a tilt of the plate holders of about 4 degrees to the horizontal.

The 4-inch lens was taken as an auxiliary; we used the square wooden tube, 19 feet

in length, originally used by Father CORTIE at Hernosand in 1914, together with the 10×8 -inch plate carriers. Study of the star-diagram showed that seven stars could be photographed by turning the plate through 45 degrees. The tube was therefore placed on its angle, large wooden V-supports being prepared to fit the tube; these rested on strong wooden trestles.

The focussing was at first done visually on Arcturus, using an eyepiece fitted with a cobalt glass (after the plate supports and object-glass had been adjusted for perpendicularity to the axis). A series of exposures was then made, the focus being varied slightly so as to cover a sufficient range. Examination of these photographs showed at once that there was serious astigmatism due to the figure of the mirror of the 16-inch cœlostet. By inserting an 8-inch stop this was reduced to a large extent, and this stop was henceforth used throughout; but the defect was of such a character that it was clear that it would be necessary to stay at Sobral and obtain comparison plates of the eclipse field in July when the sun had moved away.

The focus of the 4-inch was determined in a similar manner. The images, though superior to those of the astrographic, were not quite perfect, and here again comparison plates in July were necessary. Once the focus had been decided on, the breech end was securely screwed up to avoid any chance of subsequent movement.

A few check plates of the field near Arcturus were taken, but have not been used.

11. The following is a summary of the meteorological conditions during our stay. The barometer record was interesting in that it showed very little change from day to day, in spite of changes in the type of weather; there was, however, a very well marked semi-diurnal variation, with range of about 0.15 inch. The temperature range was fairly uniform, from a maximum of about 97° F. towards 3 p.m. to a minimum of about 75° F. at 5 a.m. The relative humidity (as shown by a hygrograph belonging to the Brazilian Commission) followed the temperature closely, varying from 30 per cent. in the afternoon to 90 per cent. in the early morning.

May is normally the last month of the rainy season at Sobral, but this year the rainfall was very scanty; there were a few afternoon showers, each ushered in by a violent gust of wind; and on May 25 there was very heavy rain, which was welcome for its moistening effect on the ground, the dust hitherto having been troublesome to the clockwork although every care had been taken to protect it. There was a fair amount of cloud in the mornings, but the afternoons and nights were clear in the majority of cases. Mt. Meruoca, 2,700 feet high, about 6 miles to the N.W., was a collector of cloud, its summit being frequently veiled in mist. In spite of its cooler climate, the summit would thus not have been a suitable eclipse station, and, in fact, nothing of the total phase of the eclipse was seen from it.

12. Although water was generally scarce, we were very fortunately situated as we enjoyed an unlimited supply of good water laid on at the house. This was of great benefit in the photographic operations. Ice was unobtainable, but by the use of earthenware water-coolers it was possible to reduce the temperature to about 75°, and by working

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 299

only at night or before dawn development of the plates was fairly easy. Formalin was used in every case to harden the films, and thereby minimise the chance of distortion due to the softening of the films by the warm solutions.

We had provided ourselves with two brands of plates, but it had become apparent from photographs taken and developed before the eclipse that one of these brands was unsuitable in the hot climate, and it was decided to use practically only one brand of plates.

In taking the experimental photographs it was noticed that the clocks and coelostats were very sensitive to wind. We had reason to fear strong gusts about the time of totality, such as had occurred in other eclipses; and as the conditions of our locality seemed to render them specially probable, protective wind screens were erected round the hut openings at every point where it was possible without interfering with the field of view. Happily dead calm prevailed at the critical time. Screens also protected all projecting parts of the telescope tubes from direct sunlight.

The performance of the 16-inch coelostat was unsatisfactory in respect of driving. There was a clearly marked oscillation of the images on the screen in a period of about 30 seconds. For this reason exposure time was shortened, so as to multiply the number of exposures in the hope that some would be near the stationary points.

13. The morning of the eclipse day was rather more cloudy than the average, and the proportion of cloud was estimated at $\frac{9}{10}$ at the time of first contact, when the sun was invisible; it appeared a few seconds later showing a very small encroachment of the moon, and there were various short intervals of sunshine during the partial phase which enabled us to place the sun's image at its assigned position on the ground glass, and to give a final adjustment to the rates of the driving clocks. As totality approached, the proportion of cloud diminished, and a large clear space reached the sun about one minute before second contact. Warnings were given 58s., 22s. and 12s. before second contact by observing the length of the disappearing crescent on the ground glass. When the crescent disappeared the word "go" was called and a metronome was started by Dr. LEOCADIO, who called out every tenth beat during totality, and the exposure times were recorded in terms of these beats. It beat 320 times in 310 seconds; allowance has been made for this rate in the recorded times. The programme arranged was carried out successfully, 19 plates being exposed in the astrographic telescope with alternate exposures of 5 and 10 seconds, and eight in the 4-inch camera with a uniform exposure of 28 seconds. The region round the sun was free from cloud, except for an interval of about a minute near the middle of totality when it was veiled by thin cloud, which prevented the photography of stars, though the inner corona remained visible to the eye and the plates exposed at this time show it and the large prominence excellently defined. The plates remained in their holders until development, which was carried out in convenient batches during the night hours of the following days, being completed by June 5.

14. No observation of contact times was made, but it is known that these times were

300 SIR F. W. DYSON, PROF. A. S. EDDINGTON AND MR. C. DAVIDSON ON A

somewhat before those calculated. As the times recorded were reckoned from second contact, it is assumed that this occurred May 28, 23h. 58m. 18s. G.M.T.

The details of the exposures are given in the following tables :—

EXPOSURES with the 13-inch Astrographic Telescope stopped to 8 inches.

Ref. No.	G.M.T. at Commencement of Exposure.				Exposure.	Plate.	Ref. No.	G.M.T. at Commencement of Exposure.				Exposure.	Plate.
	d.	h.	m.	s.	s.			d.	h.	m.	s.	s.	
1	28	23	58	23	5	O.	11	29	0	1	7	5	S.R.
2				37	10	E.	12				22	10	E.
3				57	5	E.	13				36	5	E.
4			59	11	10	S.	14				51	10	S.R.
5				30	5	S.R.	15			2	10	5	S.R.
6				45	10	S.R.	16				25	10	S.R.
7	29	0	0	4	5	S.R.	17				44	5	E.
8				19	10	E.	18				58	10	E.
9				39	5	E.	19			3	18	5	O.
10				53	10	S.R.							

EXPOSURES with the 4-inch Telescope.

Ref. No.	G.M.T. at Commencement of Exposure.				Exposure.	Plate.	Ref. No.	G.M.T. at Commencement of Exposure.				Exposure.	Plate.
	d.	h.	m.	s.	s.			d.	h.	m.	s.	s.	
1	28	23	58	21	28	S.R.	5	29	0	0	56	28	S.R.
2				59	0	S.R.	6			1	34	28	S.R.
3					38	S.R.	7			2	13	28	S.R.
4	29	0	0	17	28	S.R.	8				52	28	S.R.

In the fourth column the letter O stands for Imperial Ordinary.

E „ „ Empress.

S „ „ Sovereign.

SR „ „ Ilford Special Rapid.

With the astrographic telescope 12 stars are shown on a number of plates, and seven stars on all but three (Nos. 13, 14 and 19). Of the eight plates taken with the 4-inch lens, seven show seven stars, but No. 6, which was taken through cloud, does not show any.

The following table of temperatures, communicated by Dr. MORIZE, and converted into the Fahrenheit scale, shows how slight the fall was during totality, probably owing to the large amount of cloud in the earlier stages which checked the usual daily rise.

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 301

G.M.T.				Ther.	G.M.T.				Ther.	G.M.T.				Ther.	G.M.T.				Ther.
d.	h.	m.		°	d.	h.	m.		°	d.	h.	m.		°	d.	h.	m.		°
28	22	45		82·4	28	23	30		80·6	29	0	15		82·0	29	1	0		83·8
		23	0	84·2			45		82·4			30		82·4			15		84·2
			15	82·4	29	0	0		80·6			45		83·1			30		84·2

15. On June 7, having completed the development, we left Sobral for Fortaleza, returning on July 9 for the purpose of securing comparison plates of the eclipse field.

Before our departure we dismounted the mirrors and driving clocks which were brought into the house to avoid the exposure to dust. The telescopes and cœlostats were left *in situ*. Before removing the mirrors we marked their positions in their cells so that they could be replaced in exactly the same position.

After our return to Sobral the mirrors and clocks were remounted; the photography of the eclipse field was commenced on the morning of July 11 (civil). The difficulty of finding the field with the cœlostats was overcome by making a rough hour circle on the heads of the cœlostats out of millimetre paper.

The following is the list of exposures made on the field for comparison with the eclipse photographs :—

Astrographic Telescope.						4-inch Telescope.					
Ref. No.	Date.	G.M.T.	No. of exposures.	Duration.	Altitude.	Ref. No.	Date.	G.M.T.	No. of exposures.	Duration.	Altitude.
		h. m.		s.	°			h. m.		s.	°
11 ₁	July 10	20 5	3	5	28·9						
11 ₂		20 16	2	5	31·1						
11 ₃		20 21	1	5	32·2	14 ₁	July 13	20 7	2	25	32·4
14 ₁	July 13	20 13	3	5	33·7	14 ₂		20 16	2	20	34·3
14 ₂		20 17	2	5	34·5						
14 ₃		20 19	2	5	34·9	15 ₁	July 14	20 17	2	20	35·4
15 ₁	July 14	20 15	3	5	34·9	15 ₂		20 22	2	20	36·4
15 ₂		20 20	2	5	36·1						
15 ₃		20 23	2	5	36·6	17 ₁	July 16	20 6	3	15	34·7
17 ₁	July 16	20 2	4	3	33·8	17 ₂		20 24	2	15	38·6
17 ₂		20 15	3	3	36·6						
17 ₃		20 23	2	3	38·3						
17 ₄	July 17	20 25	2	5	38·8	18 ₁ *	July 17	19 57	3	20	33·6
18 ₁		19 50	3	4	32·8	18 ₂		20 24	2	20	39·2
18 ₂		20 1	2	4	34·4						
18 ₃		20 20	3	4	38·6						
18 ₄		20 25	2	3	39·5						

The reference numbers follow the civil dates.

* The 4-inch plate, No. 18₁, was taken through the glass (see § 17, *infra*) to facilitate the measurement, and is referred to as the scale plate.

Thermometer readings, July 10, $74^{\circ}\cdot4$; July 13, $73^{\circ}\cdot7$; July 14, $71^{\circ}\cdot9$; July 16, $72^{\circ}\cdot3$; July 17, $72^{\circ}\cdot3$.

By July 18 we had obtained a sufficient number of reference photographs. Dismantling of the instruments was commenced, and the packing was completed on July 21. We left Sobral on July 22, leaving the packing cases in the hands of Messrs. NICOLAU and CARNEIRO to be forwarded at the earliest opportunity, and arrived at Greenwich on August 25.

The observers wish to record their obligations to Mr. CHARLES BOOTH and the officers of the "Booth" Line for facilitating their journeys to and from their station at a difficult time.

PHOTOGRAPHS TAKEN WITH THE 4-INCH OBJECT GLASS.

16. These photographs were taken on 10×8 -inch plates. By suitably mounting the camera it was made possible to obtain seven stars on the photographs, viz., Nos. 2, 3, 4, 5, 6, 10 and 11 of the table in § 5. Of the eight photographs taken during the eclipse seven gave measurable images of these stars, the other plate (No. 6) taken through cloud only showing a picture of the prominences.

Plates of the same field taken under nearly similar conditions as regards altitude were taken on July 14, 15, 17 and 18 (civil date). Of these photographs, the second taken on July 14 with two exposures (referred to as 14_{2a} and 14_{2b}), two photographs taken on July 15 (referred to as 15_1 and 15_2), two on July 17 (17_1 and 17_2), and the second photograph on July 18 (18_2) were measured for comparison with the eclipse plates.

17. The micrometer at the Royal Observatory is not suitable for the direct comparison of plates of this size. It was therefore decided to measure each plate by placing, film to film upon it, another photograph of the same region reversed by being taken through the glass. A photograph for this purpose was taken on July 18. This plate is regarded merely as an intermediary between the eclipse plates and comparison plates and is referred to as the scale plate, being used simply as a scale providing points of reference. In all cases measurement was made through the glass of the scale plate, adjusted on the eclipse or comparison plate which was being measured, so that the separation of the images on the two plates did not exceed one-third of a millimetre. The plates were held together by clips which ensured contact over the whole surface. This method of measurement was found to be very convenient. Each plate was measured in two positions, being reversed through 180 degrees, and the accordance of the result showed that the method of measurement was entirely satisfactory.

The measures, both direct and reversed, were made by two measurers (Mr. DAVIDSON and Mr. FURNER), and the means taken. There was no sensible difference between the measurers, which is satisfactory, as it affords evidence of the similarity of the images on the eclipse and comparison and scale plates.

The value of the micrometer screws (both in R.A. and Decl.) is $6''\cdot25$.

18. The results of the measures are as follows :—

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 303

TABLE II.—Eclipse Plates—Scale.

No. of Star.	I.		II.		III.		IV.		V.		VII.		VIII.	
	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.
11	r -1.411	r -0.554	r -1.416	r -1.324	r +0.592	r +0.956	r +0.563	r +1.238	r +0.406	r +0.970	r -1.456	r +0.964	r -1.285	r -1.195
5	-1.048	-0.338	-1.221	-1.312	+0.756	+0.843	+0.683	+1.226	+0.468	+0.861	-1.267	+0.777	-1.152	-1.332
4	-1.216	+0.114	-1.054	-0.944	+0.979	+1.172	+0.849	+1.524	+0.721	+1.167	-1.028	+1.142	-0.927	-0.930
3	-1.237	+0.150	-1.079	-0.862	+0.958	+1.244	+0.861	+1.587	+0.733	+1.234	-1.010	+1.185	-0.897	-0.894
6	-1.342	+0.124	-1.012	-0.932	+1.052	+1.197	+0.894	+1.564	+0.798	+1.130	-0.888	+1.125	-0.838	-0.937
10	-1.289	+0.205	-0.999	-0.948	+1.157	+1.211	+0.934	+1.522	+0.864	+1.119	-0.820	+1.072	-0.768	-0.964
2	-0.789	+0.109	-0.733	-1.019	+1.256	+0.924	+1.177	+1.373	+0.995	+0.935	-0.768	+0.892	-0.585	-1.166
	-1.500*	-0.554	-1.500	-1.324	+0.500	+0.843	+0.500	+1.226	+0.400	+0.861	-1.500	+0.777	-1.300	-1.322

COMPARISON Plates—Scale.

No. of Star.	14 _a .		14 _b .		15 ₁ .		15 ₂ .		17 ₁ .		17 ₂ .		18 ₂ .	
	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.
11	r -0.478	r -0.109	r +0.967	r +1.170	r +1.098	r +1.228	r +0.725	r +0.830	r -1.073	r -1.330	r +1.242	r -0.302	r -1.188	r -1.572
5	-0.544	-0.204	+1.013	+1.192	+0.899	+1.232	+0.692	+0.938	-1.072	-1.075	+1.161	-0.224	-1.195	-1.432
4	-0.368	-0.136	+1.030	+1.249	+1.133	+1.086	+0.725	+0.854	-1.296	-1.031	+1.354	-0.281	-1.165	-1.454
3	-0.350	-0.073	+1.044	+1.305	+1.164	+1.114	+0.732	+0.893	-1.278	-1.014	+1.342	-0.261	-1.178	-1.394
6	-0.317	-0.144	+0.980	+1.319	+1.244	+1.012	+0.714	+0.824	-1.375	-1.052	+1.363	-0.390	-1.165	-1.473
10	-0.272	-0.146	+0.997	+1.327	+1.249	+0.960	+0.722	+0.831	-1.424	-1.038	+1.370	-0.423	-1.164	-1.476
2	-0.396	-0.182	+1.102	+1.289	+0.969	+1.052	+0.734	+0.941	-1.236	-0.909	+1.278	-0.328	-1.164	-1.335
	-0.552*	-0.206	+0.967	+1.170	+0.899	+0.960	+0.690	+0.824	-1.424	-1.330	+1.161	-0.423	-1.195	-1.572

* The numbers -1.500, -0.554, &c., given below the line, were taken out to make the values of Dx, Dy small and positive for arithmetical convenience.

19. The values of Dx and Dy were equated to expressions of the form

$$ax + by + c + aE_x (= Dx)$$

and

$$dx + ey + f + aE_y (= Dy),$$

where x, y are the co-ordinates of the stars given in Table I., and E_x, E_y are coefficients of the gravitational displacement.

The quantities c and f are corrections to zero, depending on the setting of the scale plate on the plate measured, a and e are differences of scale value, while b and d depend mainly on the orientation of the two plates. The quantity a denotes the deflection at unit distance (*i.e.*, 50' from the sun's centre), so that aE_x and aE_y are the deflection in R.A. and Decl. respectively of a star whose co-ordinates are x and y .

The left-hand sides of the equation for the seven stars shown are :—

No.	Right Ascension.	Declination.
11	$c - 0.160b - 1.261a - 0.587\alpha$	$f - 1.261d - 0.160e + 0.036\alpha$
5	$c - 1.107b - 0.160a - 0.557\alpha$	$f - 0.160d - 1.107e - 0.789\alpha$
4	$c + 0.472b + 0.334a - 0.186\alpha$	$f + 0.334d + 0.472e + 1.336\alpha$
3	$c + 0.360b + 0.348a - 0.222\alpha$	$f + 0.348d + 0.360e + 1.574\alpha$
6	$c + 1.099b + 0.587a + 0.080\alpha$	$f + 0.587d + 1.099e + 0.726\alpha$
10	$c + 1.321b + 0.860a + 0.158\alpha$	$f + 0.860d + 1.321e + 0.589\alpha$
2	$c - 0.328b + 1.079a + 1.540\alpha$	$f + 1.079d - 0.328e - 0.156\alpha$

20. Normal equations formed from these equations of condition are as follows :—

TABLE III.—Eclipse Plates—Right Ascension.

				I.	II.	III.	IV.	V.	VII.	VIII.
$+7.000c$	$+1.657b$	$+1.787a$	$+0.226\alpha$	$= +2.159$	$+2.986$	$+3.250$	$+2.461$	$+2.185$	$+3.263$	$+2.648$
	$+4.664$	$+2.089$	$+0.335$	$= -0.063$	$+0.986$	$+1.320$	$+0.866$	$+1.051$	$+1.464$	$+1.130$
		$+4.094$	$+2.534$	$= +1.034$	$+1.689$	$+1.866$	$+1.469$	$+1.480$	$+1.972$	$+1.723$
			$+3.142$	$= +0.712$	$+0.919$	$+0.924$	$+0.860$	$+0.844$	$+0.930$	$+0.973$
	$+4.271b$	$+1.666a$	$+0.281\alpha$	$= -0.575$	$+0.278$	$+0.550$	$+0.283$	$+0.533$	$+0.691$	$+0.502$
		$+3.683$	$+2.476$	$= +0.483$	$+0.928$	$+1.037$	$+0.841$	$+0.923$	$+1.140$	$+1.048$
			$+3.135$	$= +0.643$	$+0.823$	$+0.820$	$+0.781$	$+0.774$	$+0.826$	$+0.888$
	$+2.988a$	$+2.366\alpha$		$= +0.707$	$+0.820$	$+0.822$	$+0.731$	$+0.715$	$+0.871$	$+0.852$
			$+3.116$	$= +0.681$	$+0.805$	$+0.784$	$+0.762$	$+0.739$	$+0.780$	$+0.855$
			$+1.242\alpha$	$= +0.121$	$+0.156$	$+0.133$	$+0.183$	$+0.173$	$+0.090$	$+0.180$
			α	$= +0.098$	$+0.126$	$+0.107$	$+0.148$	$+0.140$	$+0.073$	$+0.145$
			a	$= +0.158$	$+0.174$	$+0.189$	$+0.127$	$+0.128$	$+0.233$	$+0.169$
			b	$= -0.203$	-0.011	$+0.048$	$+0.007$	$+0.042$	$+0.066$	$+0.042$

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 305

TABLE IV.—Comparison Plates—Right Ascension.

					14 _{2a} .	14 _{2b} .	15 ₁ .	15 ₂ .	17 ₁ .	17 ₂ .	18 ₂ .
+7.000c	+1.657b	+1.787a	+0.226α	=	+1.190	+0.364	+1.463	+0.214	+1.214	+0.983	+0.146
	+4.664	+2.089	+0.335	=	+0.700	+0.017	+0.992	+0.078	-0.340	+0.603	+0.083
		+4.094	+2.535	=	+0.638	+0.220	+0.499	+0.073	-0.172	+0.450	+0.085
			+3.142	=	+0.253	+0.159	-0.029	+0.037	-0.164	+0.105	+0.041
+4.271b	+1.666a	+0.281α	=	+0.418	-0.069	+0.645	+0.027	-0.627	+0.370	+0.048	
	+3.683	+2.476	=	+0.334	+0.127	+0.126	+0.018	-0.481	+0.199	+0.048	
		+3.135	=	+0.215	+0.147	-0.076	+0.030	-0.203	+0.074	+0.036	
+2.988a	+2.366α	=	+0.172	+0.154	-0.126	+0.007	-0.236	+0.055	+0.029		
	+3.116	=	+0.188	+0.152	-0.119	+0.028	-0.162	+0.050	+0.033		
	+1.242α	=	+0.052	+0.030	-0.019	+0.022	+0.025	+0.006	+0.010		
			α	=	+0.042	+0.024	-0.015	+0.018	+0.020	+0.005	+0.008
			a	=	+0.024	+0.032	-0.030	-0.012	-0.094	+0.014	+0.003
			b	=	+0.086	-0.030	+0.164	+0.012	-0.111	+0.081	+0.010

TABLE V.—Eclipse Plates—Declination.

+7.000f	+1.787d	+1.657e	+3.316α	=	+3.688	+1.927	+1.646	+1.452	+1.389	+1.718	+1.906
	+4.094	+2.089	+1.840	=	+2.200	+1.168	+0.719	+0.823	+0.555	+0.610	+0.840
		+4.664	+3.694	=	+1.860	+1.159	+1.129	+0.984	+0.874	+1.023	+1.193
			+5.784	=	+2.657	+1.681	+1.535	+1.361	+1.335	+1.545	+1.707
+3.638d	+1.666e	+0.994α	=	+1.260	+0.677	+0.299	+0.453	+0.201	+0.172	+0.354	
	+4.271	+2.908	=	+0.986	+0.702	+0.739	+0.640	+0.545	+0.616	+0.741	
		+4.212	=	+0.909	+0.768	+0.755	+0.673	+0.677	+0.731	+0.804	
+3.508e	+2.453α	=	+0.409	+0.392	+0.602	+0.431	+0.453	+0.537	+0.579		
	+3.941	=	+0.565	+0.583	+0.673	+0.549	+0.622	+0.684	+0.707		
	+2.224α	=	+0.279	+0.309	+0.252	+0.247	+0.305	+0.308	+0.302		
			α	=	+0.126	+0.139	+0.114	+0.111	+0.137	+0.139	+0.136
			e	=	+0.029	+0.015	+0.092	+0.045	+0.033	+0.056	+0.070
			d	=	+0.299	+0.141	+0.009	+0.074	+0.003	-0.016	+0.028

TABLE VI.—Comparison Plates—Declination.

+7.000f	+1.787d	+1.657e	+3.316α	=	+0.446	+0.661	+0.964	+0.343	+1.861	+0.752	+0.868
	+4.094	+2.089	+1.840	=	+0.060	+0.420	-0.156	+0.140	+1.038	+0.041	+0.476
		+4.664	+3.694	=	+0.202	+0.394	-0.203	-0.117	+0.526	-0.110	+0.122
			+5.784	=	+0.380	+0.482	+0.220	+0.044	+1.004	+0.296	+0.419
+3.638d	+1.666e	+0.994α	=	-0.054	+0.251	-0.402	+0.053	+0.563	+0.151	+0.255	
	+4.271	+2.908	=	+0.096	+0.237	-0.431	-0.198	+0.085	-0.288	-0.084	
		+4.212	=	+0.168	+0.169	-0.237	-0.119	+0.122	-0.060	+0.008	
+3.508e	+2.453α	=	+0.121	+0.122	-0.247	-0.222	-0.173	-0.219	-0.201		
	+3.941	=	+0.183	+0.100	-0.127	-0.133	-0.032	-0.019	-0.062		
	+2.224α	=	+0.098	+0.015	+0.046	+0.022	+0.089	+0.134	+0.079		
			α	=	+0.044	+0.007	+0.021	+0.010	+0.040	+0.060	+0.036
			e	=	+0.004	+0.030	-0.085	-0.070	-0.077	-0.104	-0.082
			d	=	-0.028	+0.054	-0.077	+0.044	+0.179	-0.010	+0.098

21. The values of α are collected in Table VII :—

TABLE VII.

Right Ascension.		Declination.	
Eclipse — Scale.	Comparison — Scale.	Eclipse — Scale.	Comparison — Scale.
r +0·098 +0·126 +0·107 +0·148 +0·140 +0·073 +0·145	r +0·042 +0·024 −0·015 +0·018 +0·020 +0·005 +0·008	r +0·126 +0·139 +0·114 +0·111 +0·137 +0·139 +0·136	r +0·044 +0·007 +0·021 +0·010 +0·040 +0·060 +0·036
Mean +0·120	+0·015	+0·129	+0·031

By subtracting the α of the comparison plates the scale plate is eliminated, and we derive from right ascensions $\alpha = +0^{\circ}\cdot105$ and from declinations $\alpha = +0^{\circ}\cdot098$.

Reference to the normal equations shows that the declination result is of double the weight of that from the right ascensions.

Thus

$$\alpha = +0^{\circ}\cdot100 = +0''\cdot625.$$

This is at a distance $50'$ from the sun's centre. At the time of the eclipse the sun's radius was $15'\cdot8$; thus the deflection at the limb is $1''\cdot98$.

The range in the values of α is attributable to the errors inherent to the star images of the different plates, and cannot be reduced by further measurement. The mean values $+0^{\circ}\cdot015$ and $0^{\circ}\cdot031$ arise from the errors in the intermediary scale plate.

22. The probable error of the result judging from the accordance of the separate determinations is about 6 per cent. It is desirable to consider carefully the possibility of systematic error. The eclipse and comparison photographs were taken under precisely similar instrumental conditions, but there is the difference that the eclipse photographs were taken on the day of May 29, and the comparison photographs on nights between July 14 and July 18. A very satisfactory feature of the photographs is the essential similarity of the star images on the two sets of photographs.

The satisfactory accordance of the eclipse and comparison plates is shown by a study of the plate constants. The following corrections for differential refraction and aberration are calculated from the times and dates of exposure.

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 307

	<i>a.</i>	<i>e.</i>	<i>b.</i>	<i>d.</i>
	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>
Eclipse plates	+0·240	+0·168	+0·062	+0·062
Scale plate	+0·423	+0·207	+0·096	+0·096
Comparison 14 _{2a}	+0·409	+0·207	+0·091	+0·091
„ 14 _{2b}	+0·409	+0·207	+0·091	+0·091
„ 15 ₁	+0·390	+0·207	+0·087	+0·087
„ 15 ₂	+0·370	+0·202	+0·087	+0·087
„ 17 ₁	+0·399	+0·207	+0·091	+0·091
„ 17 ₂	+0·337	+0·202	+0·077	+0·077
„ 18 ₂	+0·327	+0·202	+0·072	+0·072

When these are applied to the values of the constants found from the normal equations, we find the following values of the scale of the several photographs and their orientation relative to the scale plate :—

		Scale Value.		Orientation.		Adopted Scale Orientation.
		From <i>x</i> .	From <i>y</i> .	From <i>x</i> .	From <i>y</i> .	
		<i>r</i>	<i>r</i>			<i>r</i>
Eclipse	I.	−0·025	−0·010	−0·237	−0·265	0·000 −0·251
„	II.	−0·009	−0·024	−0·045	−0·107	0·000 −0·076
„	III.	+0·006	+0·053	+0·014	+0·025	0·000 +0·020
„	IV.	−0·056	+0·006	−0·027	−0·040	0·000 −0·034
„	V.	−0·055	−0·006	+0·008	+0·031	0·000 +0·020
„	VII.	+0·050	+0·017	+0·032	+0·050	0·000 +0·041
„	VIII.	−0·014	+0·031	+0·008	+0·006	0·000 +0·007
Comparison	14 _{2a}	+0·010	+0·004	+0·081	+0·033	+0·013 +0·057
„	14 _{2b}	+0·008	+0·030	−0·035	−0·049	+0·013 −0·042
„	15 ₁	−0·063	−0·085	+0·155	+0·086	−0·084 +0·120
„	15 ₂	−0·065	−0·075	+0·003	−0·035	−0·084 −0·016
„	17 ₁	−0·118	−0·077	−0·116	−0·174	−0·084 −0·145
„	17 ₂	−0·072	−0·109	+0·062	+0·029	−0·084 +0·046
„	18 ₂	−0·093	−0·087	−0·014	−0·074	−0·084 −0·044

The agreement in the scale values obtained from *x* and *y* is satisfactory. There appears to be a small difference in the orientations as derived from the two directions in the comparison plates. This is, however, of small importance in the determination of α . There is a difference of scale value from July 15–18 shown in both co-ordinates. For the purpose of exhibiting the gravitational displacements, residuals have been computed using adopted values for the scale and orientation given above, along with the calculated corrections for differential refraction and aberration. This has the advantage of reducing the number of constants employed in the reduction of the plates, and lessens the possibility of masking any discordances, though greater irregularities necessarily appear when four arbitrary constants instead of six are used in the reduction

of each plate. The quantities are converted from revolutions to seconds of arc, as the more familiar unit facilitates judgment of the results.

TABLE VIII.—Comparison of the Eclipse and Comparison Photographs with the Scale Plate, after Correction for Differential Refraction and Aberration, Orientation and Change of Scale.

No. of Star.	I.	II.	III.	IV.	V.	VII.	VIII.	Mean.
ECLIPSE Plates—Right Ascension.								
	"	"	"	"	"	"	"	"
11	−0.18	−0.51	−0.46	−0.07	−0.04	−0.72	−0.43	−0.34
5	−0.45	−0.81	−0.38	−0.58	−0.60	−0.36	−0.62	−0.54
4	+0.08	+0.11	−0.08	−0.11	−0.11	−0.16	−0.18	−0.06
3	−0.23	−0.11	−0.19	−0.05	−0.02	−0.02	−0.01	−0.09
6	−0.14	+0.23	−0.09	−0.11	−0.13	+0.13	−0.08	−0.03
10	+0.17	+0.06	+0.14	−0.18	−0.11	+0.14	−0.01	+0.03
2	+0.75	+1.03	+1.06	+1.09	+1.01	+0.98	+1.30	+1.03
ECLIPSE PLATES—Declination.								
	"	"	"	"	"	"	"	"
11	0.00	−0.08	−0.03	+0.02	+0.17	+0.16	+0.01	+0.03
5	−0.38	−0.54	−0.61	−0.30	−0.39	−0.73	−0.81	−0.54
4	+1.19	+1.04	+1.03	+0.98	+1.11	+1.19	+1.24	+1.11
3	+1.42	+1.58	+1.50	+1.39	+1.55	+1.49	+1.49	+1.49
6	+0.65	+0.79	+1.01	+0.97	+0.71	+0.95	+1.01	+0.87
10	+0.62	+0.46	+1.03	+0.54	+0.56	+0.58	+0.74	+0.65
2	+0.01	+0.25	−0.40	−0.09	−0.22	−0.14	−0.17	−0.11
	14 _{2a} .	14 _{2b} .	15 ₁ .	15 ₂ .	17 ₁ .	17 ₂ .	18 ₂ .	Mean.
COMPARISON Plates—Right Ascension.								
	"	"	"	"	"	"	"	"
11	−0.19	−0.24	−0.23	−0.28	+0.11	−0.19	−0.02	−0.15
5	−0.42	+0.16	−0.36	−0.32	−0.24	−0.33	−0.26	−0.25
4	−0.01	+0.03	−0.01	+0.05	−0.04	+0.23	+0.08	+0.05
3	+0.14	+0.09	+0.28	+0.10	−0.03	+0.21	−0.01	+0.11
6	+0.02	−0.18	+0.26	+0.06	+0.13	+0.03	+0.14	+0.07
10	+0.17	−0.06	+0.20	+0.18	+0.13	−0.02	+0.15	+0.11
2	+0.31	+0.18	−0.16	+0.22	−0.04	+0.08	−0.06	+0.08
COMPARISON Plates—Declination.								
	"	"	"	"	"	"	"	"
11	−0.07	+0.08	−0.26	−0.04	−0.26	−0.18	−0.16	−0.13
5	−0.23	−0.03	+0.03	0.00	−0.19	+0.03	−0.20	−0.08
4	+0.23	+0.05	+0.29	+0.18	+0.45	+0.53	+0.23	+0.28
3	+0.64	+0.41	+0.42	+0.36	+0.48	+0.60	+0.54	+0.49
6	+0.22	+0.36	+0.33	+0.26	+0.41	+0.21	+0.32	+0.30
10	+0.28	+0.32	+0.31	+0.36	+0.36	+0.15	+0.29	+0.30
2	+0.25	+0.14	+0.18	+0.21	+0.09	−0.03	+0.27	+0.16

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 309

Subtracting the results of the comparison plates, so as to eliminate the errors arising from the intermediary scale plate we find for the displacements of the different stars, as compared with those as given by EINSTEIN'S Theory, with value $1''.75$ at the sun's limb :—

No. of Star.	Displacement in Right Ascension.		Displacement in Declination.	
	Observed.	Calculated	Observed.	Calculated.
	"	"	"	"
11	−0.19	−0.32	+0.16	+0.02
5	−0.29	−0.31	−0.46	−0.43
4	−0.11	−0.10	+0.83	+0.74
3	−0.20	−0.12	+1.00	+0.87
6	+0.10	+0.04	+0.57	+0.40
10	−0.08	+0.09	+0.35	+0.32
2	+0.95	+0.85	−0.27	−0.09

PHOTOGRAPHS TAKEN WITH THE ASTROGRAPHIC OBJECT GLASS.

23. As stated above these photographs were taken with the astrographic object glass stopped down to 8 inches, mounted in a steel tube and fed by a 16-inch cœlost. From many years' experience with the object glass at Greenwich it is certain that, when the object glass is mounted in a steel tube, the change of scale over a range of temperature of 10° F. should be insignificant, and the definition should be very good. It was realised that this high standard would not be obtained with the glass used in conjunction with the cœlost. taken to Brazil, but nevertheless the results shown when the plates were developed were very disappointing. The images were diffused and apparently out of focus, although on the night of May 27 the focus was good.* Worse still, this change was temporary, for without any change in the adjustments, the instrument had returned to focus when the comparison plates were taken in July.

These changes must be attributed to the effect of the sun's heat on the mirror, but it is difficult to say whether this caused a real change of scale in the resulting photographs or merely blurred the images.

The photographs were measured in the astrographic duplex micrometer, the eclipse photographs being directly compared with the comparison plates taken in July. All

* The following note made at the time is quoted in full :—" May 30, 3 a.m., four of the astrographic plates were developed, and when dry examined. It was found that there had been a serious change of focus, so that, while the stars were shown, the definition was spoilt. This change of focus can only be attributed to the unequal expansion of the mirror through the sun's heat. The readings of the focussing scale were checked next day, but were found unaltered at 11.0 mm. It seems doubtful whether much can be got from these plates."

the stars shown were measured. They were reduced by the same method as that employed for the "4-inch" photographs. With the exception of plates Nos. 15 and 16, taken through clouds, the stars numbered 3, 4, 5, 6, 10, 11 and 12 are shown on all the plates; the fainter stars 2, 7, 8 and 9 are sometimes shown, but No. 1, which is very near the sun, is always drowned in the corona. These plates were only measured in declination, as the right ascensions were of little weight.

24. In the following table is given the value of α , the constant of the gravitational displacement, as calculated from the measures; the apparent difference of scale e between the eclipse and comparison plates; d the difference of orientation of the plates given by the measures of y , and depending on the adjustment of the plates in the measuring machine.

TABLE IX.
($1'' = 12'' \cdot 3$).

No. of Eclipse Plate.	Ref. No. of Comparison Plate.	No. of Stars.	Values of d , e , α in Revolutions at 50' Distance.			α at Sun's Limb in Arc.
			d .	e .	α .	
1	18 ₄	7	r +0.051	r +0.089	r +0.033	" +1.28
2	18 ₄	11	-0.009	+0.059	+0.025	+0.97
3	18 ₄	8	-0.074	+0.101	+0.028	+1.09
4	18 ₄	11	-0.168	+0.091	+0.033	+1.28
5	11 ₃	10	+0.094	+0.076	+0.025	+0.97
6	11 ₃	11	+0.186	+0.082	+0.021	+0.82
{ 7	14 ₃	12	+0.006	+0.119	0.000	0.00
	18 ₃	7	-0.054	+0.166	0.000	0.00
8	14 ₃	10	+0.093	+0.064	+0.021	+0.82
9	17 ₄	7	-0.096	+0.129	+0.008	+0.31
10	17 ₄	10	+0.090	+0.045	+0.026	+1.01
11	11 ₁	10	+0.073	+0.061	+0.032	+1.24
{ 12	11 ₁	11	-0.009	+0.102	+0.049	+1.91
{ 12	17 ₂	7	-0.102	+0.114	+0.019	+0.74
15	15 ₃	6	+0.111	+0.036	+0.018	+0.70
16	15 ₃	7	-0.002	+0.037	+0.018	+0.70
17	17 ₂	8	-0.022	+0.109	+0.012	+0.47
18	17 ₂	7	+0.045	0.000	+0.030	+1.17
Mean				+0.082	+0.022	+0.86

Thus the mean value of α obtained from all the astrographic plates is $0'' \cdot 86$, a figure considerably less than that obtained from the 4-inch photographs.

25. Reference to the diagram shows that the measurement of displacement depends essentially on the position of the stars Nos. 3 and 4 relative to 5 on one side and 6 and 10 on the other. These are all bright stars, and in this respect their images are

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 311

more comparable than are the images of the fainter stars. The measures of these stars are given in the following table :—

No. of Eclipse Plate.	Measured Values of Dy for Stars Nos.—					No. of Eclipse Plate.	Measured Values of Dy for Stars Nos.—				
	5	4	3	6	10		5	4	3	6	10
	r	r	r	r	r		r	r	r	r	r
1	−0·051	+0·175	+0·169	+0·201	+0·235	9	−0·059	+0·121	+0·109	+0·205	+0·180
2	+0·558	+0·656	+0·724	+0·668	+0·702	10	+0·033	+0·270	+0·188	+0·258	+0·280
3	+0·124	+0·285	+0·286	+0·274	+0·355	11	+0·025	+0·215	+0·210	+0·233	+0·274
4	+0·111	+0·222	+0·247	+0·231	+0·167	12	−0·068	+0·144	+0·124	+0·160	+0·167
5	+0·034	+0·228	+0·232	+0·218	+0·308	15	−0·038	+0·138	+0·107	+0·172	—
6	+0·164	+0·488	+0·478	+0·557	+0·637	16	−0·050	+0·076	+0·046	+0·127	+0·073
7	−0·051	+0·156	+0·162	+0·250	+0·279	17	−0·071	+0·104	+0·081	+0·186	+0·164
8	+0·108	+0·330	+0·314	+0·376	+0·397	18	+0·016	+0·092	+0·109	+0·099	+0·084

The equations given by these stars are

$$-0·160d - 1·107e - 0·789a + f = Dy_5 \quad (1)$$

$$+0·334d + 0·472e + 1·336a + f = Dy_4 \quad (2)$$

$$+0·348d + 0·360e + 1·574a + f = Dy_3 \quad (3)$$

$$+0·587d + 1·099e + 0·726a + f = Dy_6 \quad (4)$$

$$+0·860d + 1·321e + 0·589a + f = Dy_{10} \quad (5)$$

The mean of (4) and (5) added to (1) gives

$$+0·564d + 0·103e - 0·131a + 2f = Dy_5 + \frac{1}{2}(Dy_6 + Dy_{10}).$$

While the sum of (2) and (3) gives,

$$+0·682d + 0·832e + 2·910a + 2f = Dy_3 + Dy_4.$$

Subtracting these we get

$$3·041a + 0·729e + 0·118d = Dy_3 + Dy_4 - Dy_5 - \frac{1}{2}(Dy_6 + Dy_{10}).$$

This equation has a small coefficient for e and a very small one for d .

Calculating the quantities on the right-hand side, assuming e to be the same for all the plates, and substituting the values of d from the previous table, we find :—

r		r	
$a + 0·240e = +0·056$	1	$a + 0·240e = +0·035$	9
$a + 0·240e = +0·049$	2	$a + 0·240e = +0·048$	10
$a + 0·240e = +0·047$	3	$a + 0·240e = +0·045$	11
$a + 0·240e = +0·059$	4	$a + 0·240e = +0·059$	12
$a + 0·240e = +0·050$	5	$a + 0·283e = +0·026$	15
$a + 0·240e = +0·059$	6	$a + 0·240e = +0·024$	16
$a + 0·240e = +0·036$	7	$a + 0·240e = +0·028$	17
$a + 0·240e = +0·046$	8	$a + 0·240e = +0·029$	18

In photograph No. 15, star 10 is not shown, and the equation is slightly modified. It may also be noticed that the values are somewhat smaller for Nos. 15 to 18.

The means of the 16 photographs treated in this manner give

$$\alpha + 243e = + 0^{\circ} \cdot 0435,$$

or with the value of the scale $0^{\circ} \cdot 082$ from the previous table

$$\alpha = + 0^{\circ} \cdot 024 = 0'' \cdot 93 \text{ at the limb.}$$

It may be noticed that the change of scale arising from differences of refraction and aberration is $0^{\circ} \cdot 020$. If this value of e be taken instead of $0^{\circ} \cdot 082$ we obtain

$$\alpha = + 0^{\circ} \cdot 039 = + 1'' \cdot 52 \text{ at the sun's limb.}$$

The equations on p. 311 were also solved by least squares for each plate. There is a considerable range in the deduced values of α , as is to be expected when α and e are determined independently for each plate. The mean result for α is $0'' \cdot 99$, or very nearly the same as that already found.

The photographs taken with the astrographic telescope support those obtained by the "4-inch" to the extent that they show considerable outward deflection, but for the reasons already given are of much less weight.

IV. THE EXPEDITION TO PRINCEPE.

(*Observers, Prof. A. S. EDDINGTON and Mr. E. T. COTTINGHAM.*)

26. The expedition left Liverpool on the "Anselm" on March 8, and travelled in company with the Sobral expedition as far as Madeira. It was necessary to wait there until April 9, when the journey was continued on the "Portugal," belonging to the Companhia Nacional de Navegação. The expedition landed at the small port of S. Antonio in the Isle of Principe on April 23.

Vice-Admiral CAMPOS RODRIGUES and Dr. F. OOM of the National Observatory, Lisbon, had kindly given us introductions, and everything possible was done by those on the island for the success of the work and the comfort of the observers. We were met on board by the Acting Administrator Sr. VASCONCÉLOS, Sr. CARNEIRO, President of the Association of Planters, and Sr. GRAGEIRA, representing the Sociedade d'Agricultura Colonial, who made all necessary arrangements. The Portuguese Government dispensed with any customs examination of the baggage.

27. Principe is a small island belonging to Portugal, situated just north of the equator in the Gulf of Guinea, about 120 miles from the African coast. The extreme length and breadth are about 10 miles and 6 miles. Near the centre mountains rise to a height of 2500 feet, which generally attract heavy masses of cloud. Except for a certain amount of virgin forest, the island is covered with cocoa plantations. The

climate is very moist, but not unhealthy. The vegetation is luxuriant, and the scenery is extremely beautiful. We arrived near the end of the rainy season, but the *gravana*, a dry wind, set in about May 10, and from then onwards no rain fell except on the morning of the eclipse.

We were advised that the prospects of clear sky at the end of May were not very good, but that the best chance was on the north and west of the island. After inspecting two other sites on the property of the Sociedade d'Agricultura Colonial, we fixed on Roça Sundy, the headquarters of Sr. CARNEIRO's chief plantation. We were Sr. CARNEIRO's guests during our whole visit, and used freely his ample resources of labour and material at Sundy. We learnt later that he had postponed a visit to Europe in order to entertain us. We were also greatly indebted to his manager at Sundy, Sr. ATALAYA, with whom we lived for five weeks; his help and attention were invaluable. Mr. WRIGHT and Mr. LEWIS of the Cable Station kindly assisted us as interpreters when necessary.

Sundy is situated in the north-west of the island overlooking the sea at a height of 500 feet, and as far as possible from the cloud-gathering peaks. Our telescope was erected in a small walled enclosure adjoining the house, from which the ground sloped steeply down to the sea in the direction of the sun at eclipse. On the other side it was sheltered by a building. The approximate position was latitude $1^{\circ} 40' N.$, longitude $29m. 32s. E.$

28. The baggage was brought to Sundy on April 28 mainly by tram, but with a break of about a kilometre, where it had to be transported through the wood by native carriers. After a week spent on the preparations, we returned to S. Antonio for the week, May 6–13, as it was undesirable to unpack the mirror so early in the damp climate. On our return to Sundy the installation and adjustments were soon completed, and the first check plates were taken on May 16. Meanwhile the *gravana* had begun, which, although there is no rain, is generally accompanied by increased cloud. There were, however, some days of clear sky, and the nights were usually clear.

The *coelostat* was mounted on a stone pier built for the purpose. The clock weight fell into a pit below the clock deep enough to allow a run of 36 minutes without rewinding. Care was taken to use a particular part of the *coelostat*-sector, considered to be the most perfect, in photographing the eclipse and the check field. The telescope (Oxford astrographic object-glass, see p. 295) rested on wooden V's near the two ends, the V's being supported on packing-cases; the one at the breech-end could be moved laterally to allow of different declination settings, and was marked with an approximate declination scale. A series of exposures of one second was made on a bright star to test whether there was any shake of the telescope after inserting the plate: no shake was detected even when the exposure was made immediately; but as a safeguard for the eclipse photographs a full second was allowed to elapse before beginning the exposure. The exposure was made by moving a cardboard screen

unconnected with the instrument. The telescope pointed slightly downwards, and the tube was turned so as to give the right orientation to the plate, the lines of declination being two or three degrees inclined to the horizontal. A canvas screen was arranged to protect the tube and object-glass from the direct radiation of the sun.

The adjustments call for little comment. In view of the purpose of the observations, it was desirable to adjust the tilt of the object-glass and plate with special care. It was also important that the setting on the field should be nearly exact. The sun appeared on the eclipse day in sufficient time to allow of the setting being made by means of the solar image; but arrangements had been tested by which the correct field would have been obtained if it had been cloudy up to totality.* The telescope was focussed by trial photographs of stars, and owing to the uniform temperature of the island the focus was unchanged for day observations.

The object-glass was stopped down to 8 inches for the eclipse photographs and for all check and comparison photographs used in the reductions.

29. The days preceding the eclipse were very cloudy. On the morning of May 29 there was a very heavy thunderstorm from about 10 a.m. to 11.30 a.m.—a remarkable occurrence at that time of year. The sun then appeared for a few minutes, but the clouds gathered again. About half-an-hour before totality the crescent sun was glimpsed occasionally, and by 1.55 it could be seen continuously through drifting cloud. The calculated time of totality was from 2h. 13m. 5s. to 2h. 18m. 7s. G.M.T. Exposures were made according to the prepared programme, and 16 plates were obtained. Mr. COTTINGHAM gave the exposures and attended to the driving mechanism, and Prof. EDDINGTON changed the dark slides. It appears from the results that the cloud must have thinned considerably during the last third of totality, and some star images were shown on the later plates. The cloudier plates give very fine photographs of a remarkable prominence which was on the limb of the sun.

A few minutes after totality the sun was in a perfectly clear sky, but the clearance did not last long. It seems likely that the break-up of the clouds was due to the eclipse itself, as it was noticed that the sky usually cleared at sunset.

It had been intended to complete all the measurements of the photographs on the spot; but owing to a strike of the steamship company it was necessary to return by the first boat, if we were not to be marooned on the island for several months. By the intervention of the Administrator berths, commandeered by the Portuguese Government, were secured for us on the crowded steamer. We left Principe on June 12, and after transhipping at Lisbon, reached Liverpool on July 14.

30. The following is a list of the photographs, including the comparison photographs kindly taken for us by Mr. F. A. BELLAMY at Oxford, before the instrument was dismantled. All the eclipse photographs are given, though only W and X furnished

* The method depended on setting the cross-wires of the theodolite (attached to the *coelostat*) on a terrestrial mark, and then starting the clock at a particular instant.

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 315

results. Of the other series, only the exposures actually used in the reductions are given.

LIST of Plates.

Check Field (R.A. 14h. 12m. 47s., Declination $+20^{\circ} 30'$)

Ref.	Place.	Date.	Loc. Sid. T.	Exp.	Approx. Z.D.	Bar.	Ther.	Plate.
		1919.	h. m. s.	s.	°	m.	°	
a_1	Oxford	January 16	12 55 10	60	35	29·64	37·0	S.
b_1	"	January 17	13 10 40	60	34	29·83	35·3	S.
c_1	"	"	13 54 55	60	31	29·83	35·3	S.
d_1	"	"	14 9 25	60	31	29·83	35·3	S.
e_1	"	January 23	13 13 30	60	33	30·45	29·0	S.
			G.M.T.					
g_1	Principe	May 22	12 25 40	40	43	29·45	76·5	S.R.
r_1	"	"	12 31 20	40	45	29·45	76·5	S.R.
s_2	"	"	12 37 50	80	46	29·45	76·5	S.R.
v_1	"	May 25	12 22 20	40	45	29·45	76·5	S.S.
w_1	"	"	12 26 20	40	46	29·45	76·5	S.S.

NOTES.

Column 1.—The letter is marked on the original plates (preserved at Cambridge Observatory). The number refers to the exposure, disregarding exposures taken without the 8-inch stop.

Column 2.—The co-ordinates of Oxford Observatory are 5m. 3s. W., $51^{\circ} 46'$ N., and of the site at Principe, 29m. 32s. E., $1^{\circ} 40'$ N.

Column 4.—The mid-instant of the exposure is given. Times for check plates at Principe were only noted roughly. Times for the eclipse plates are deduced from the calculated time of totality, the interval from the end of one exposure to the beginning of the next being assumed uniform.

Column 7.—Readings at Principe were taken with an aneroid recording instrument, and therefore automatically reduced to the latitude of England. The barometer during our visit was practically constant except for a regular semi-diurnal wave of amplitude about 0·05 in.

Column 9.—Brand of Plate: S. = Imperial Sovereign, S.S. = Imperial Special Sensitive, S.R. = Ilford Special Rapid, E. = Ilford Empress. Backed plates were used at Principe.

Eclipse Field (R.A. 4h. 19m. 30s., Declination $+21^{\circ} 43'$)

Ref.	Place.	Date.	Loc. Sid. T.			Exp.	Approx. Z.D.	Bar.	Ther.	Plate.
		1919.	h.	m.	s.	s.	°	m.	°	
D ₁	Oxford	January 16	3	58	1	5	30	29·65	39·0	S.
G ₁	"	January 22	4	4	39	5	30	30·30	31·0	S.
H ₁	"	"	4	34	28	5	30	30·30	31·0	S.
I ₂	"	"	4	48	46	10	31	30·30	31·0	S.
K ₂	"	February 9	4	45	24	10	30	30·48	24·5	S.
			G.M.T.							
K'	Principe	May 29	2	13	9	5	46	29·45	77·0	S.R.
L	"	"	2	13	28	10	46	29·45	77·0	S.R.
M	"	"	2	13	46	3	46	29·45	77·0	S.R.
N	"	"	2	14	1	5	46	29·45	77·0	E.
O	"	"	2	14	20	10	46	29·45	77·0	S.S.
P	"	"	2	14	44	15	46	29·45	77·0	S.S.
Q	"	"	2	15	6	5	46	29·45	77·0	S.R.
R	"	"	2	15	30	20	46	29·45	77·0	S.R.
S	"	"	2	15	53	3	46	29·45	77·0	S.S.
T	"	"	2	16	13	15	46	29·45	77·0	E.
U	"	"	2	16	37	10	46	29·45	77·0	S.R.
V	"	"	2	16	56	5	46	29·45	77·0	S.S.
W	"	"	2	17	15	10	46	29·45	77·0	S.
X	"	"	2	17	33	3	46	29·45	77·0	S.R.
Y	"	"	2	17	47	2	46	29·45	77·0	S.R.
Z	"	"	2	18	1	2	46	29·45	77·0	S.R.

NOTES.

Columns 1 to 9. See previous page.

The large proportion of Ilford Special Rapid plates used at the eclipse was due to the fact that experience in developing the check plates showed that these suffered less than the others from the high temperature of the water (78° F.). Ice was generally available for the check plates through the kindness of Sr. GRAGEIRA; but the supply failed after the eclipse, and formalin was used to harden the films. This was unsatisfactory except for the I.S.R. plates, and so plates P, S, T, W were brought home undeveloped. The developing at Principe was done at night, and the drying was accelerated by use of alcohol.

The use of an 8-inch stop in front of the object-glass was suggested to us by Mr. DAVIDSON, who showed that a great improvement of the images resulted; it was originally intended, however, to use the full aperture for part of totality. Early measures of check plates made at Principe soon convinced us that the results from the full aperture were greatly inferior, and we decided to rely entirely on the 8-inch aperture.

The Check Plates.

31. In addition to the eclipse field, a check field was photographed both at Oxford and at Principe. The field chosen included Arcturus, so that it was easily found with the cœlostat. Its declination was nearly the same as that of the eclipse field, and it was photographed at the same altitude at Principe in order that any systematic error, due to imperfections of the cœlostat mirror or other causes, might affect both sets of plates equally. The primary purpose was thus to check the possibility of systematic error arising from the different conditions of observation at Oxford and Principe, and from possible changes in the object-glass during transit. Unlike the Sobral expedition, we were not able to take comparison photographs of the eclipse field at Principe, because for us the eclipse occurred in the afternoon, and it would be many months before the field could be photographed in the same position in the sky before dawn. The check plates were therefore specially important for us.

As events turned out the check plates were important for another purpose, viz., to determine the difference of scale at Oxford and Principe. As shown in the report of the Sobral expedition, it is not necessary to know the scale of the eclipse photographs, since the reductions can be arranged so as to eliminate the unknown scale. If, however, a trustworthy scale is known and used in the reductions, the equations for the deflection have considerably greater weight, and the result depends on the measurement of a larger displacement. On surveying the meagre material which the clouds permitted us to obtain, it was evident that we must adopt the latter course; and accordingly the first step was to obtain from the check plates a determination of the scale of the Principe photographs.

32. All the measures were made by Prof. EDDINGTON with the Cambridge measuring machine.* An Oxford and a Principe plate were placed film to film so that the images of corresponding stars nearly coincided—this was possible because the Oxford plates were taken direct, and the Principe plates by reflection in the cœlostat mirror.

The small differences Δx and Δy , in the sense Principe—Oxford, were then measured for each star. Eight settings were made on each image; for half of them the field was rotated through 180 degrees by the reversion prism. Five pairs of plates were measured, and the measures are given in Table XI.

* 'Monthly Notices, R.A.S.,' vol. LXI, p. 444.

TABLE XI.—Check Plates, Measures.

Star.	Approx. Co-ords.		q_1-a_1 .		w_1-b_1 .		s_2-c_1 .		r_1-d_1 .		v_1-e_1 .	
	x .	y .	Δx .	Δy .	Δx .	Δy .	Δx .	Δy .	Δx .	Δy .	Δx .	Δy .
1	1.41	20.31	4346	7180	3199	4259	6012	7375	3921	8796	5435	4399
2	5.89	12.74	3865	6405	3394	4129	4922	6132	3039	7440	5978	4170
4	9.46	11.13	3640	5932	3408	4118	4369	5366	2638	6776	5966	4441
5	12.00	6.84	3311	5590	—	—	3831	4752	1938	6156	—	4314
6	12.80	27.33	5415	6561	3192	5140	7689	5925	5379	7580	5032	5794
7	13.75	13.78	4076	5630	3496	4290	4891	4805	3101	6461	5906	4826
8	15.50	24.38	5125	6300	—	—	—	—	—	—	5139	5412
10	20.13	10.49	3965	4940	3679	4505	4656	3568	2866	5370	6398	5229
11	20.81	0.93	2874	4352	3876	3759	2845	2815	1238	4758	7268	4482
12	22.91	6.23	3685	4436	3931	4158	4039	2738	2270	4551	6765	5076
13	26.46	8.96	4222	4288	4045	4326	4724	2232	2720	4120	6836	5561

The unit for x and y is 5 millimetres, which is approximately equal to $5'$. The differences Δx , Δy are given in units of the fifth place of decimals = $0''.003$. The centre of the plate is near $x = 14$, $y = 14$.

Plate-constants were then calculated in the usual way, by the formulæ

$$\Delta x = ax + by + c$$

$$\Delta y = dx + ey + f$$

These were applied, and the residuals $\Delta_1 x$, $\Delta_1 y$ converted into arc are as follows :—

TABLE XII.—Check Plates, Residuals.

Star.	q_1-a_1 .		w_1-b_1 .		s_2-c_1 .		r_1-d_1 .		v_1-e_1 .		Mean.	
	$\Delta_1 x$.	$\Delta_1 y$.	$\Delta_1 x$.	$\Delta_1 y$.	$\Delta_1 x$.	$\Delta_1 y$.	$\Delta_1 x$.	$\Delta_1 y$.	$\Delta_1 x$.	$\Delta_1 y$.	$\Delta_1 x$.	$\Delta_1 y$.
1	—	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—	—
4	—	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	—	—	—	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—	—
12	—	—	—	—	—	—	—	—	—	—	—	—
13	—	—	—	—	—	—	—	—	—	—	—	—
1	—0.02	—0.02	+0.29	—0.34	+0.02	—0.07	—0.03	+0.22	+0.49	+0.01	+0.15	—0.04
2	+0.39	+0.15	+0.16	+0.14	+0.69	0.00	+0.69	—0.29	+0.10	—0.23	+0.41	—0.05
4	—0.14	—0.04	—0.16	+0.09	—0.38	—0.12	—0.02	—0.37	—0.54	+0.12	—0.25	—0.06
5	—0.08	+0.35	—	—	+0.25	+0.19	—0.21	—0.21	—	—0.01	—0.01	+0.08
6	—0.06	—0.10	—0.28	+0.27	—0.09	+0.14	—0.10	+0.12	+0.15	+0.49	—0.08	+0.18
7	—0.06	—0.28	—0.10	—0.16	—0.74	—0.09	—0.31	+0.02	—0.39	—0.12	—0.32	—0.13
8	—0.30	+0.34	—	—	—	—	—	—	—0.38	—0.68	—0.34	—0.17
10	—0.02	—0.10	—0.21	+0.52	—0.15	+0.16	+0.08	+0.25	—0.08	+0.34	—0.08	+0.23
11	—0.46	—0.01	—0.13	—0.22	—0.13	+0.11	—0.13	+0.71	+0.30	—0.28	—0.11	+0.06
12	+0.16	—0.14	+0.13	—0.04	+0.19	—0.06	+0.17	—0.09	—0.13	+0.08	+0.10	—0.05
13	+0.59	—0.12	+0.32	—0.26	+0.34	—0.25	—0.13	—0.38	+0.48	+0.28	+0.32	—0.15

The mean residual without regard to sign is $\pm 0''.21$, from which the probable error of a determination of Δx or Δy is $\pm 0''.22$.

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 319

Star 7 is much the brightest. Stars 1, 6, 11, 13 are rather bright. Stars 2, 4, 10, 12 are fainter and more comfortable to measure. Stars 5 and 8 are very faint. Arcturus is on the plates but is much too bright to measure. No measures have been rejected.

The determination of the deflection on the eclipse plates is based on the declinations (y), and the last column of Table XII. shows that on the check plates the y -comparisons are free from any serious systematic error.

Star 7 is of particular interest; its position near the centre of the field corresponds to that of κ_1 , κ_2 Tauri in the eclipse field, from which the greatest deflection is expected. The images (which are not quite round) have the same characteristic shape. Further, the brightness of No. 7 corresponds with, but exaggerates, the brightness of κ_1 Tauri which is the brightest star in the eclipse field. It is therefore a valuable check to find that its systematic error in declination is insignificant compared with the displacement (of the order of $1''$) afterwards found for κ_1 and κ_2 Tauri.

The systematic errors in right ascension are larger (probably through imperfect driving of the clock). They may affect the displacement indirectly through the orientation constant, but with much reduced effect. Allowing for this reduction in importance there appears to be nothing to trouble about.

The primary purpose of the check plates is thus fulfilled. They show that photographs of a check field of stars taken at Oxford and Principe show none of the displacements which are exhibited by the photographs of the eclipse field taken under precisely similar instrumental conditions. The inference is that the displacements in the latter case can only be attributed to presence of the eclipsed sun in the field.

33. We turn now to the differences of scale between Oxford and Principe, which are given by the plate-constants a , b , d , e determined from the measures. As determined, these include the effects of differential refraction and aberration. The latter corrections were calculated for each plate by the usual formulæ and applied, so as to determine the corrected plate-constants, a' , b' , d' , e' free from differential refraction and aberration. Due allowance was made for the change in the coefficient of refraction owing to the difference of barometer and temperature (about 40°) between Oxford and Principe. The results are as follows (in units of the fifth place of decimals):—

TABLE XIII.—Check Plates, Plate-Constants.

Comparison.	Uncorrected.				Corrected.				
	a .	b .	d .	e .	a' .	b' .	d' .	e' .	$b'+d'$.
$q_1 - a_1$	+32.7	+101.0	— 87.8	+58.2	+32.7	+ 98.4	— 90.4	+32.1	+ 8.0
$w_1 - b_1$	+26.2	— 16.0	+ 25.9	+53.6	+30.4	— 22.5	+ 19.4	+31.4	— 3.1
$s_2 - c_1$	+31.5	+192.5	—173.5	+64.8	+35.8	+182.6	—183.4	+42.1	— 0.8
$r_1 - d_1$	+28.2	+165.0	—146.8	+69.8	+32.1	+157.8	—154.0	+45.0	+ 3.8
$v_1 - e_1$	+21.6	— 76.2	+ 70.6	+61.4	+25.2	— 80.5	+ 66.3	+35.7	—14.2
Mean					+31.2	—	—	+37.3	— 1.3

The sign of the results shows that the scale of the photographs is larger at Principe than at Oxford; in fact the focus must have been set about 1.2 mm. further out (apart from any change of length compensated by expansion of the photographic plates). As the error in focussing was probably not more than 0.5 mm., the greater part of this shift must be due to the focal length of the lens combination increasing with temperature more rapidly than the linear expansion of the glass.

If the only difference were a change of focal length, we should have $a' = e'$. There is a fairly strong indication that e' is greater than a' . This is no doubt due to a change in the definition caused by the coelostat mirror or by a shift of the object-glass lenses on the journey; and, as it will presumably affect the eclipse plates in the same way, it is best to adopt the values of a' and e' as determined, rather than to take a mean. In so doing we shall at any rate not exaggerate the displacement, which depends mainly on the y -measures and is reduced by adopting too large a value of e' .*

The difference $b' - d'$ merely gives the relative orientation of the two plates as placed face to face. The sum $b' + d'$ practically vanishes, as it should do. However, for consistency we adopt the small value found.

From the internal discordances of our determination of e' (the most important of these constants) the probable error of the mean is ± 2.1 . This, as shown later, will cause a probable error of our final determination of the deflection, reduced to the limb of the sun, of amount $\pm 0''.14$, affecting all determinations systematically. Errors in the other constants have much smaller influence.

The Eclipse Plates.

34. The eclipse plates from K to S show no star images. After that the cloud lightened somewhat, and some images appear on the remaining plates. The sky was never clear and nothing fainter than $5'.5$ is shown. The cloud was variable in different parts of the plate, so that the brightness of the images varies erratically and the diffusion is also variable.

In order to obtain results of any weight the stars 4 and 3 (κ_1 and κ_2 Tauri), which theoretically should be strongly displaced, must be shown. They appear on all plates from T to Z, and being near the centre of the field have good images. They are relatively rather faint on plate U, but are bright on the other plates. The appearance of the remaining stars is as follows:—

Plate T.	6 bright; 10 faint.
Plate U.	6, 10 very bright; 11 faint.
Plate V.	6 bright; 10 fair.
Plate W.	5, 6 good; 10 diffused.
Plate X.	5, 6, 11 good.
Plate Y.	5, 6, 11 faint, diffused; 12 very faint.
Plate Z.	5, 6, 11 faint, diffused.

* It happens that it is also reduced, but to a less extent, by using too small a value of a' .

The possibility of a determination of deflection practically depends on the appearance of star 5. The relative displacement of 5 and 3 is on EINSTEIN'S theory, $1''.2$ in the y -co-ordinate. Further, the x -measures of 5 are needed for a really good determination of the orientation. Star 11 can scarcely take its place. It is true that the relative displacement is then $0''.8$; but the orientation affects this with a much larger factor, and the orientation is badly determined in the absence of star 5.

Accordingly plates W and X are the only ones likely to give a trustworthy result. X is somewhat the better plate of the two.* Measures have been made of the faint diffused images on plates Y and Z; but, as might have been expected, they are hopelessly discordant and cannot be reconciled by any adopted value of the deflection.

35. We give the measures of plates X and W in detail. Both comparisons of X were measured at Principe a few days after the eclipse. Plate W, which was not developed until after the return of the expedition, was measured at Cambridge on August 22-23.†

Plate X.

(1) Comparison with Oxford Plate G_1 .

The differential refraction for all the eclipse plates is

$$a = -46.5, \quad b, d = +8.2, \quad e = -27.0$$

the differential aberration being zero.

For the comparison plate G_1

$$a = -19.1, \quad b, d = +0.7, \quad e = -28.3.$$

Hence for $X - G_1$

$$a = -27.4, \quad b, d = +7.5, \quad e = +1.3.$$

* Plate X has also the merit of a short exposure, 3s. We should mistrust the x -measures of a long exposure with variable cloud and imperfect guiding, because there is nothing to show that the images of the different stars are formed at the same time.

† Of the comparisons of check plates, $w_1 - b_1$ was measured on August 20, and the others about the end of September. Previous measures had been made at Principe with three earlier check plates taken on the night of May 16; but a slight change of adjustment of tilt was made the following day (thereafter it remained unaltered until the eclipse), and the small change of focus allowed for in the comparisons. These furnished a provisional scale which was used to obtain preliminary results. Afterwards the measurement of check plates was undertaken in a more systematic way, using later plates about which no doubt could arise, and giving the results printed above. No change of any importance was found; the final value for the deflection at the limb was reduced by $0''.4$ compared with the provisional value, but this was mainly due to the adoption of separate values of a' and e' instead of adopting the mean, and to recalculation of the differential refraction and aberration.

To these must be added the terms representing change of scale, determined from the check plates (Table XIII.), viz.,

$$a = + 31 \cdot 2, \quad b, d = - 0 \cdot 6, \quad e = + 37 \cdot 3.$$

Hence the whole difference $X - G_1$ is given by

$$a = + 3 \cdot 8, \quad b, d = + 6 \cdot 9, \quad e = + 38 \cdot 6.$$

The first step is to take the measured differences Δx , Δy , and take out the parts $ax + by$, $dx + ey$, due to these terms, leaving the corrected differences $\Delta_1 x$, $\Delta_1 y$.

$\Delta_1 x$ and $\Delta_1 y$ contain (1) the Einstein displacement, if any, and (2) the unknown relative orientation of the plates giving rise to terms of the form, $\Delta x = + \theta y$, $\Delta y = - \theta x$. These two parts could be separated by a least-squares solution, but in view of the poor quality of the material it seems better to adopt a method which keeps a better check on possible discordances and shows more clearly what is happening. The Einstein displacement in x is small, and we might perhaps neglect it altogether in determining θ from the x -measures. However, it is clear from preliminary trials that a displacement exists—whether the half or the full Einstein displacement. Hence if we take out three-quarters of the full Einstein displacement ($\frac{3}{4}E_x$) we divide the already slight effect by 4, and at the same time deal fairly between the two hypotheses.* The residuals $\Delta_2 x$ result.

From the equations $\Delta_2 x = c + \theta y$ we determine by least squares the orientation θ , which is found to be $+ 163$. Removing the term $163y$ we obtain the residuals $\Delta_3 x$.

Turning to $\Delta_1 y$, we correct for the orientation by taking out the term $-163x$, leaving $\Delta_3 y$. These values should agree for all the stars, except for the displacement and the accidental error.

Denoting the value of the displacement at $50'$ (or 10 réseau-intervals) from the centre of the sun by κ , the y -displacements of the various stars will be $\kappa\alpha_y$, where α_y has the values tabulated below. We can therefore obtain κ by solving by least-squares the equations

$$\Delta_3 y = f + \kappa\alpha_y.$$

The radius of the sun during the eclipse was $15' \cdot 78$. Hence the full Einstein displacement of $1'' \cdot 75$ corresponds to $0'' \cdot 55$ at $50'$ distance, or, in our units of $0'' \cdot 003$, $\kappa = 184$. It is easily seen that the value is somewhere near this, and it is therefore easier and more instructive to take out $E_y = 184\alpha_y$, and determine the correction to κ from the residuals $\Delta_4 y$. We also remove the mean of $\Delta_4 y$ obtaining the final residuals.

The normal equations corresponding to equations of condition

$$\text{residual} = \delta f + \alpha_y \delta \kappa$$

* The smaller the displacement provisionally assumed for x , the larger is the displacement ultimately found from y (see p. 327).

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 323

are found to be

$$5\delta f + 2\cdot83\delta\kappa = -1$$

$$2\cdot83\delta f + 4\cdot83\delta\kappa = +64$$

whence

$$3\cdot23\delta\kappa = +64,$$

$$\delta\kappa = +20.$$

An increase of 20 on 184 corresponds to an increase of $0''\cdot19$ on $1''\cdot75$. Hence the resulting deflection at the limb is $1''\cdot94$.

Since the full deflection is indicated we complete the results for x by taking out the remaining $\frac{1}{4}E_x$, obtaining Δ_4x , and then tabulate the residuals from the mean values -5942 .

The successive steps are shown below:—

Star.	x .	Δx .	$3\cdot8x$.	$6\cdot9y$.	Δ_1x .	$\frac{3}{4}E_x$.	Δ_2x .	$+163y$.	Δ_3x .	Δ_4x .	Resid. (unit = $0''\cdot003$).
11	1·39	—3916	5	86	—4007	—76	—3931	2021	—5952	—5927	+ 15
5	12·40	—5518	47	20	—5585	—79	—5506	478	—5984	—5958	— 16
4	17·34	—2869	66	129	—3064	—54	—3010	3051	—6061	—6043	—101
3	17·48	—2924	66	121	—3111	—69	—3042	2869	—5911	—5888	+ 54
6	19·87	—1568	75	172	—1815	+ 3	—1818	4075	—5893	—5894	+ 48

Star.	y .	Δy .	$6\cdot9x$.	$38\cdot6y$.	Δ_1y .	$-163x$.	Δ_3y .	E_y .	Δ_4y .	α_y .	Resid.
11	12·40	6398	10	479	5909	— 227	6136	+ 6	6130	+0·03	+ 5
5	2·93	4121	86	113	3922	—2021	5943	—127	6070	—0·69	— 55
4	18·72	4512	120	722	3670	—2826	6496	+234	6262	+1·27	+137
3	17·60	4236	121	679	3436	—2849	6285	+272	6013	+1·48	—112
6	24·99	4148	137	965	3046	—3239	6285	+136	6149	+0·74	+ 24

(2) Comparison with Oxford Plate H_1 .

The reductions are similar and are given in a rather more condensed form below. The theoretical plate constants are

$$a = +3\cdot8, \quad b, d = +8\cdot3, \quad e = +38\cdot6.$$

Star.	Δx .	Δ_1x .	Δ_2x .	$+10y$.	Δ_3x .	Δ_4x .	Resid.
11	7290	7182	7258	124	7134	7159	+235
5	6751	6680	6759	29	6730	6756	—168
4	7126	6905	6959	187	6772	6790	—134
3	7320	7108	7177	176	7001	7024	+100
6	7429	7147	7144	250	6894	6893	— 31

Star.	$\Delta y.$	$\Delta_1 y.$	$-10x.$	$\Delta_3 y.$	$E_y.$	$\Delta_4 y.$	Resid.
11	1586	1095	- 14	1109	+ 6	1103	+172
5	858	642	-124	766	-127	893	- 38
4	1881	1015	-173	1188	+234	954	+ 23
3	1785	961	-175	1136	+272	864	- 67
6	1909	779	-199	978	+136	842	- 89

The normal equations are

$$5\delta f + 2\cdot83\delta\kappa = +1$$

$$2\cdot83\delta f + 4\cdot83\delta\kappa = -105$$

whence

$$3\cdot23\delta\kappa = -105,$$

$$\delta\kappa = -33.$$

The corresponding deflection at the limb is

$$1''\cdot75 - 0''\cdot31 = 1''\cdot44.$$

Plate W.

Although the exposure was only 10 seconds the images have jumped in R.A., so that the appearance is dumb-bell shaped. They are, however, symmetrical, so that fair measures of x can be made; the y measures on which the result chiefly depends are unaffected. Star 10 is very diffused in R.A.

(1) Comparison with Oxford Plate D₁.

Theoretical plate-constants

$$a = +4\cdot9, \quad b, d = +6\cdot5, \quad e = +39\cdot7.$$

Star.	$x.$	$\Delta x.$	$\Delta_1 x.$	$\frac{3}{4}E_x.$	$\Delta_2 x.$	$+91y.$	$\Delta_3 x.$	$\Delta_4 x.$	Resid.
5	12·40	2450	2370	-79	2449	267	2182	2208	+ 40
4	17·34	3948	3741	-54	3795	1704	2091	2109	- 59
3	17·48	3834	3634	-69	3703	1602	2101	2124	- 44
6	19·87	4525	4266	+ 3	4263	2275	1988	1987	-181
10	22·60	5199	4911	+17	4894	2476	2418	2412	+244

Star.	$y.$	$\Delta y.$	$\Delta_1 y.$	$-91x.$	$\Delta_3 y.$	$E_y.$	$\Delta_4 y.$	$\alpha_y.$	Resid.
5	2·93	5320	5123	-1128	6251	-127	6378	-0·69	+ 70
4	18·72	5745	4889	-1578	6467	+234	6233	+1·27	- 75
3	17·60	5911	5098	-1591	6689	+272	6417	+1·48	+109
6	24·99	5628	4507	-1808	6315	+136	6179	+0·74	-129
10	27·21	5616	4389	-2057	6446	+114	6332	+0·62	+ 24

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 325

Normal equations

$$5\delta f + 3\cdot42\delta\kappa = -1$$

$$3\cdot42\delta f + 5\cdot21\delta\kappa = -62$$

whence

$$2\cdot87\delta\kappa = -61$$

$$\delta\kappa = -21.$$

Hence deflection at the limb is

$$1''\cdot75 - 0''\cdot20 = 1''\cdot55.$$

(2) Comparison with Oxford Plate I₂.

Theoretical plate constants

$$a = +4\cdot0, \quad b, d = +9\cdot1, \quad e = +38\cdot8.$$

Star.	$\Delta x.$	$\Delta_1 x.$	$\Delta_2 x.$	$-30y.$	$\Delta_3 x.$	$\Delta_4 x.$	Resid.
5	5050	4973	5052	- 88	5140	5166	+ 46
4	4732	4493	4547	-562	5109	5127	+ 7
3	4622	4392	4461	-528	4989	5012	-108
6	4635	4329	4326	-750	5076	5075	- 45
10	4764	4426	4409	-816	5225	5219	+ 90

Star.	$\Delta y.$	$\Delta_1 y.$	$+30x.$	$\Delta_3 y.$	$E_y.$	$\Delta_4 y.$	Resid.
5	-6824	-7051	372	-7423	-127	-7296	- 15
4	-5751	-6635	520	-7155	+234	-7389	-108
3	-5609	-6451	524	-6975	+272	-7247	+ 34
6	-5425	-6576	596	-7172	+136	-7308	- 27
10	-5109	-6371	678	-7049	+114	-7163	+118

Normal equations

$$5\delta f + 3\cdot42\delta\kappa = +2$$

$$3\cdot42\delta f + 5\cdot21\delta\kappa = -24$$

whence

$$2\cdot87\delta\kappa = -25,$$

$$\delta\kappa = -9.$$

Hence deflection at the limb is

$$1''\cdot75 - 0''\cdot08 = 1''\cdot67.$$

Plate U.

Comparison with Oxford Plate K₂.

Since Plate U shows some good images it has been examined, although owing to the absence of star 8 the weight is small. The measures were made at Principe.

Theoretical plate-constants

$$a = +2.8, \quad b, d = +8.9, \quad e = +37.7.$$

Star.	x .	Δx .	$\Delta_1 x$.	$+240y$.	E_x .	$\Delta_4 x$.	Resid.
11	1.39	2905	2791	2976	-101	- 84	-147
4	17.34	4508	4292	4493	- 72	-129	-192
3	17.48	4626	4420	4224	- 92	+288	+225
6	19.87	6270	5992	5998	+ 4	- 10	- 73
10	22.60	7110	6805	6530	+ 23	+252	+189

Star.	x .	Δy .	$\Delta_1 y$.	$-240x$.	E_y .	$\Delta_4 y$.	Resid.
11	12.40	9026	8547	- 334	+ 6	8875	- 94
4	18.72	5846	4986	-4162	+234	8914	- 55
3	17.60	5985	5165	-4195	+272	9089	+120
6	24.99	5458	4339	-4769	+136	8972	+ 3
10	27.21	4911	3684	-5424	+114	8994	+ 25

In this case it is not possible to determine the orientation with sufficient accuracy from the x -measures; the value here applied is an arbitrary preliminary value. We accordingly make a least-squares solution from both x - and y -residuals to determine the correction to the orientation, $\delta\theta$, as well as δc , δf and $\delta\kappa$.

The result is

$$\delta\theta = +2, \quad \delta\kappa = +121.$$

This gives the deflection

$$2''.90.$$

The probable error is, however, $\pm 0''.87$, so that the result is practically worthless. Further, it is much more likely to be affected by systematic error than the previous results.

The large probable error is partly due to the large residuals which are greater than in the previous measures; in particular star 3 is unduly faint. If the same accuracy had been obtained, the theoretical weight would have been half that of plates W and X;

but having regard to possible systematic error, probably a quarter weight would more nearly represent the true value.

This determination is ignored in the subsequent discussion.

36. It is easy to calculate the effects of any errors in the adopted scale, orientation, &c., on the final result (deflection at the limb). We give some illustrations.

An error in the adopted scale of y of 10 units (in the fifth place of decimals) would lead to an error $0''.68$ in the result from either plate. Thus the probable error ± 2.1 in the determination of e' gives a probable error $\pm 0''.14$ in the final result; or, if we adopted the largest (rather discordant) value found for e' instead of the mean, we should reduce the result by $0''.52$.

An error of 10 units in the orientation gives an error in the result of $0''.45$ for plate X, and $0''.22$ for Plate W. It is therefore of less importance, and further it is not likely to be systematic.

Errors in the measurement of x only affect the result through the orientation. For Plate X, a probable error of $\pm 0''.20$ in the x -measures would give an error ± 4.0 in the orientation, leading to an error $\pm 0''.18$ in the result; whereas an error of the same magnitude in the y measures gives directly an error $\pm 0''.35$ in the result. For Plate W, the probable error of $\pm 0''.20$ in x gives an error ± 3.5 in the orientation and $\pm 0''.08$ in the result, compared with $\pm 0''.38$ for similar inaccuracy in y . It is particularly fortunate that the x -measures are so unimportant for Plate W, because, as already mentioned, the images trailed on that plate.

Finally, it will be remembered that in order not to commit ourselves to the Einstein hypothesis prematurely we neglected the correction $\frac{1}{4}E_x$ in determining the orientation. This will make a difference of $0''.029$ in the results from Plate W and $0''.092$ from Plate X. The effect is that the deduced deflection needs to be decreased, and the mean correction $-0''.06$ should be applied to the mean result obtained, or rather, to make the adopted deflection for x consistent with the deduced value from y , the correction needed is $-0''.04$.

Discussion of the Results.

37. The four determinations from the two eclipse plates are

X — G	$1''.94$
X — H	$1''.44$
W — D	$1''.55$
W — I	$1''.67$

giving a mean of

$$1''.65.$$

They evidently agree with EINSTEIN'S predicted value $1''.75$.

The residuals* in the separate comparisons reduced to arc are as follows. They do not appear to show any special peculiarities.

Star.	<i>x</i> residuals.					<i>y</i> residuals.				
	G.	H.	D.	I.	Mean.	G.	H.	D.	I.	Mean.
	"	"	"	"	"	"	"	"	"	"
11	+0.04	+0.70	—	—	—	+0.01	+0.52	—	—	—
5	−0.05	−0.50	+0.12	+0.14	−0.07	−0.16	−0.11	+0.21	−0.04	−0.02
4	−0.30	−0.40	−0.18	+0.02	−0.21	+0.41	+0.07	−0.22	−0.32	−0.02
3	+0.16	+0.30	−0.13	−0.32	0.00	−0.34	−0.20	+0.33	+0.10	−0.03
6	+0.14	−0.09	−0.54	−0.13	−0.16	+0.07	−0.27	−0.39	−0.08	−0.17
10	—	—	+0.73	+0.27	—	—	—	+0.07	+0.35	—

The average *y*-residual is $\pm 0''.22$, which gives a probable error for *y* of $\pm 0''.21$. It is satisfactory that this agrees so nearly with the probable error ($\pm 0''.22$) of the check plates, showing that the images are of about the same degree of difficulty and therefore presumably comparable. The probable error of *x* is $\pm 0''.25$, but we are not so much concerned with this.

The weight of the determination of $\delta\kappa$ is about 3 (strictly 3.23 for Plate X and 2.87 for Plate W). The probable error of κ is therefore $\pm 0''.12$, which corresponds to a probable error of $\pm 0''.38$ in the final values of the deflection.

As the four determinations involve only two eclipse plates and are not wholly independent, and further small accidental errors may arise through inaccurate determination of the orientation, the probable error of our mean result will be about $\pm 0''.25$. There is further the error of $\pm 0''.14$ affecting all four results equally, arising from the determination of scale. Taking this into account, and including the small correction $-0''.04$ previously mentioned, our result may be written

$$1''.61 \pm 0''.30.$$

It will be seen that the error deduced in this way from the residuals is considerably larger than at first seemed likely from the accordance of the four results. Nevertheless the accuracy seems sufficient to give a fairly trustworthy confirmation of EINSTEIN's theory, and to render the half-deflection at least very improbable.

38. It remains to consider the question of systematic error. The results obtained with a similar instrument at Sobral are considered to be largely vitiated by systematic

* The residuals refer to the theoretical deflection $1''.75$, not the deduced deflections.

errors. What ground then have we—apart from the agreement with the far superior determination with the 4-inch lens at Sobral—for thinking that the present results are more trustworthy?

At first sight everything is in favour of the Sobral astrographic plates. There are 12 stars shown against 5, and the images though far from perfect are probably superior to the Principe images. The multiplicity of plates is less important, since it is mainly a question of systematic error. Against this must be set the fact that the five stars shown on Plates W and X include all the most essential stars; stars 3 and 5 give the extreme range of deflection, and there is no great gain in including extra stars which play a passive part. Further, the gain of nearly two extra magnitudes at Sobral must have meant over-exposure for the brighter stars, which happen to be the really important ones; and this would tend to accentuate systematic errors, whilst rendering the defects of the images less easily recognised by the measurer. Perhaps, therefore, the cloud was not so unkind to us after all.

Another important difference is made by the use of the extraneous determination of scale for the Principe reductions. Granting its validity, it reduces very considerably both accidental and systematic errors. The weight of the determination from the five stars with known scale is more than 50 per cent. greater than the weight from the 12 stars with unknown scale. Its effect as regards systematic error may be seen as follows. Knowing the scale, the greatest relative deflection to be measured amounts to $1''.2$ on EINSTEIN'S theory; but if the scale is unknown and must be eliminated, this is reduced to $0''.67$. As we wish to distinguish between the full deflection and the half deflection, we must take half these quantities. Evidently with poor images it is much more hopeful to look for a difference of $0''.6$ than for $0''.3$. It is, of course, impossible to assign any precise limit to the possible systematic error in interpretation of the images by the measurer; but we feel fairly confident that the former figure is well outside possibility.

A check against systematic error in our discussion is provided by the check plates, as already shown. Its efficacy depends on the similarity of the images on the check plates and eclipse plates at Principe. Both sets are fainter than the Oxford images with which they are compared, the former owing to the imperfect driving of the cœlostæt, which made it impossible to secure longer exposures, the latter owing to cloud. Both sets have a faint wing in declination, but this is separated by a slight gap from the true images, and, at least on the plates measured, the wing can be distinguished and ignored. The images on Plates W and X are not unduly diffused except for No. 10 on Plate W. Difference in quality between the eclipse images and the Principe check images is not noticeable, and is certainly far less than the difference between the latter and the Oxford images; and, seeing that the latter comparison gives no systematic error in y , it seems fair to assume that the comparison of the eclipse plates is free from systematic error.

The writer must confess to a change of view with regard to the desirability of using

an extraneous determination of scale. In considering the programme it had seemed too risky a proceeding, and it was thought that a self-contained determination would receive more confidence. But this opinion has been modified by the very special circumstances at Principe; and it is now difficult to see that any valid objection can be brought against the use of the scale.

The temperature at Principe was remarkably uniform and the extreme range probably did not exceed 4° during our visit—including day and night, warm season and cold season. The temperature ranged generally from $77\frac{1}{2}^{\circ}$ to $79\frac{1}{2}^{\circ}$ in the rainy season, and about 1° colder in the cool gravana. All the check plates and eclipse plates were taken within a degree of the same temperature, and there was, of course, no perceptible fall of temperature preceding totality. To avoid any alteration of scale in the daytime the telescope tube and object-glass were shaded from direct solar radiation by a canvas screen; but even this was scarcely necessary, for the clouds before totality provided a still more efficient screen, and the feeble rays which penetrated could not have done any mischief. A heating of the mirror by the sun's rays could scarcely have produced a true alteration of scale though it might have done harm by altering the definition; the cloud protected us from any trouble of this kind. At the Oxford end of the comparison the scale is evidently the same for both sets of plates, since they were both taken at night and intermingled as regards date.

It thus appears that the check scale is legitimately applicable to the eclipse plates. But the method may not be so satisfactory at future eclipses, since the particular circumstances at Principe are not likely to be reproduced. As regards other sources of systematic error, our chief guarantee lies in the comparatively large amount of the deflection to be measured, and the test satisfied by the check plates that photographs of another field under similar conditions show no deflections comparable with those here found.

V. GENERAL CONCLUSIONS.

39. In summarising the results of the two expeditions, the greatest weight must be attached to those obtained with the 4-inch lens at Sobral. From the superiority of the images and the larger scale of the photographs it was recognised that these would prove to be much the most trustworthy. Further, the agreement of the results derived independently from the right ascensions and declinations, and the accordance of the residuals of the individual stars (p. 308) provides a more satisfactory check on the results than was possible for the other instruments.

These plates gave

From declinations	$1''.94$
From right ascensions	$2''.06$

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 331

The result from declinations is about twice the weight of that from right ascensions, so that the mean result is

$$1''.98$$

with a probable error of about $\pm 0''.12$.

The Principe observations were generally interfered with by cloud. The unfavourable circumstances were perhaps partly compensated by the advantage of the extremely uniform temperature of the island. The deflection obtained was

$$1''.61.$$

The probable error is about $\pm 0''.30$, so that the result has much less weight than the preceding.

Both of these point to the full deflection $1''.75$ of EINSTEIN'S generalised relativity theory, the Sobral results definitely, and the Principe results perhaps with some uncertainty. There remain the Sobral astrographic plates which gave the deflection

$$0''.93$$

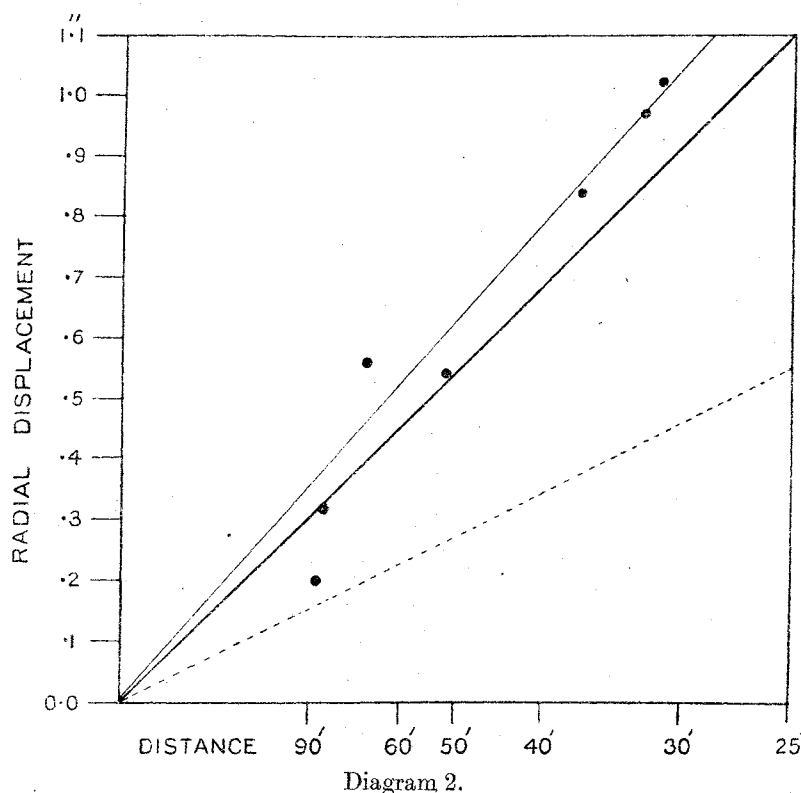
discordant by an amount much beyond the limits of its accidental error. For the reasons already described at length not much weight is attached to this determination.

It has been assumed that the displacement is inversely proportional to the distance from the sun's centre, since all theories agree on this, and indeed it seems clear from considerations of dimensions that a displacement, if due to gravitation, must follow this law. From the results with the 4-inch lens, some kind of test of the law is possible though it is necessarily only rough. The evidence is summarised in the following table and diagram, which show the radial displacement of the individual stars (mean from all the plates) plotted against the reciprocal of the distance from the centre. The displacement according to EINSTEIN'S theory is indicated by the heavy line, according to the Newtonian law by the dotted line, and from these observations by the thin line.

RADIAL Displacement of Individual Stars.

Star.	Calculation.	Observation.
	"	"
11	0.32	0.20
10	0.33	0.32
6	0.40	0.56
5	0.53	0.54
4	0.75	0.84
2	0.85	0.97
3	0.88	1.02

Thus the results of the expeditions to Sobral and Principe can leave little doubt that a deflection of light takes place in the neighbourhood of the sun and that it is of the amount demanded by EINSTEIN'S generalised theory of relativity, as attributable to the sun's gravitational field. But the observation is of such interest that it will probably be considered desirable to repeat it at future eclipses. The unusually favourable conditions of the 1919 eclipse will not recur, and it will be necessary to photograph fainter stars, and these will probably be at a greater distance from the sun.

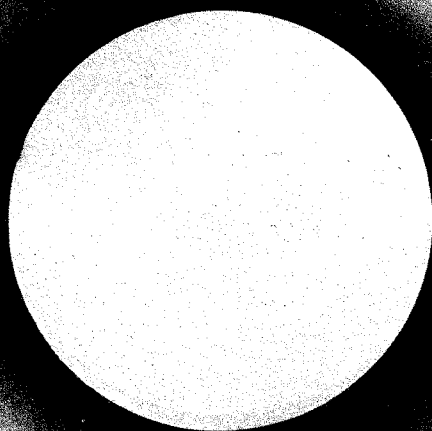


This *can* be done with such telescopes as the astrographic with the object-glass stopped down to 8 inches, if photographs of the same high quality are obtained as in regular stellar work. It will probably be best to discard the use of coelostat mirrors. These are of great convenience for photographs of the corona and spectroscopic observations, but for work of precision of the high order required, it is undesirable to introduce complications, which can be avoided, into the optical train. It would seem that some form of equatorial mounting (such as that employed in the Eclipse Expeditions of the Lick Observatory) is desirable.

In conclusion, it is a pleasure to record the great assistance given to the Expeditions from many quarters. Reference has been made in the course of the paper to some of these. Especial thanks are due to the Brazilian Government for the hospitality and facilities accorded to the observers in Sobral. They were made guests of the

Dyson and others.

Phil. Trans., A, vol. 220, Plate 1.



DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 333

Government, who provided them with transport, accommodation and labour. Dr. MORIZE, Director of the Rio Observatory, acting on behalf of the Brazilian Government, made most complete arrangements for the Expedition, and in this way contributed materially to its success.

On behalf of the Principe Expedition, special thanks are due to Sr. JERONYMO CARNEIRO, who most hospitably entertained the observers and provided for all their requirements, and to Sr. ATALAYA, whose help and friendship were of the greatest service to the observers in their isolated station.

We gratefully acknowledge the loan for more than six months of the astrographic object-glass of the Oxford University Observatory. We are also indebted to Mr. BELLAMY for the check plates he obtained in January and February.

Thanks are due to the Royal Irish Academy for the loan of the 4-inch object-glass and 8-inch coelostat.

As stated above, the expeditions were arranged by the Joint Permanent Eclipse Committee with funds allocated by the Government Grant Committee.

[In Plate 1 is given a half-tone reproduction of one of the negatives taken with the 4-inch lens at Sobral. This shows the position of the stars, and, as far as possible in a reproduction of this kind, the character of the images, as there has been no retouching.

A number of photographic prints have been made and applications for these from astronomers, who wish to assure themselves of the quality of the photographs, will be considered and as far as possible acceded to.]
