Forkeppni 2019

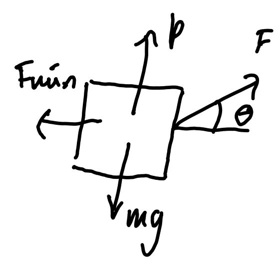
Lausnir

Krossar

1. C: $K = \frac{1}{2}mv^2$ og $[K] = [\frac{1}{2}mv^2] = [m]\cdot[v]^2 = \frac{kg\, m^2}{s^2}$.

2. D: $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{\left(\frac{100}{3,6}\right) - 0}{2,5} = 11\ m/s^2$.

3. A:



Leið 1: $F_{nún} = F\cos\theta = 43\ N$.

Leið 2: $F_{nún} = \mu p$ og $p + F\sin\theta = mg$ svo

$$F_{nún} = \mu(mg - F\sin\theta) = 43\ N.$$

4. E: $E_{fyrir} + W_{nún} = E_{eftir}$ svo $mgh - \mu mgd = 0$

svo $d = \frac{h}{\mu} = 25\ m$.

5.c:

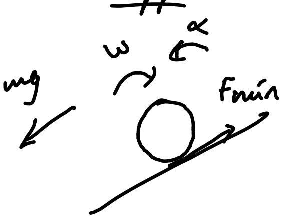
Hvað er hann lengi að detta?

$$\Delta y = \frac{1}{2} g t_B^2 \quad \text{þ.a.} \quad t_B = \sqrt{\frac{2\Delta y}{g}}$$

en þá er heildartíminn $T = 2t_B = \sqrt{\frac{8\Delta y}{g}} = 5{,}7s$

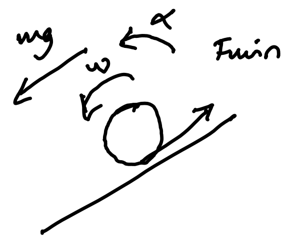
6.d:

upp:



upp

Niður:



upp

7.B:

Leið 1: $R_{eq} = \frac{1}{x} + \frac{1}{1/x} = x + \frac{1}{x}$

og $V = IR_{eq}$ gefur $1 = 2\left(x + \frac{1}{x}\right)$

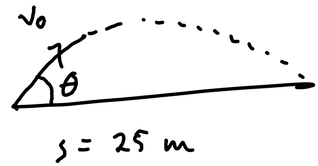
þ.e. $x^2 - \frac{1}{2}x + 1 = 0$ sem hefur lausn

$$x = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4}}{2} = \frac{1}{4} \pm \frac{\sqrt{15}}{4} i \quad \text{svo} \quad x + \frac{1}{x} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\,\Omega$$

Leið 2: $R_{eq} = \frac{V}{I} = \frac{1}{2} = 0{,}5\,\Omega$.

8.D:

$$\begin{pmatrix} s \\ 0 \end{pmatrix} = \begin{pmatrix} v_0\cos\theta\, t \\ v_0\sin\theta\, t - \frac{1}{2}gt^2 \end{pmatrix}$$



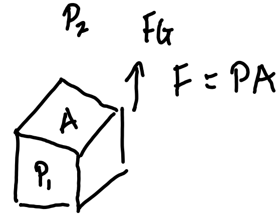
efri jafnan gefur $t = \frac{s}{v_0\cos\theta}$

neðri jafnan gefur: $0 = \frac{s\cdot\sin\theta}{\cos\theta} - \frac{gs^2}{2v_0^2\cos^2\theta}$

þ.a. $2\sin\theta\cos\theta = \sin(2\theta) = \frac{gs}{v_0^2}$

en þá er $2\theta = \begin{cases} \arcsin\left(\frac{gs}{v_0^2}\right) \\ 180^\circ - \arcsin\left(\frac{gs}{v_0^2}\right) \end{cases} = \begin{cases} 38^\circ \\ 142^\circ \end{cases}$ þ.a. $\theta = \begin{cases} 19^\circ \\ 71^\circ \end{cases}$

9.E:



$$F_G + F_1 = F_2$$

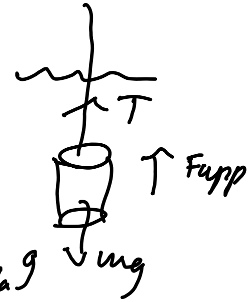
svo $F_G = F_2 - F_1 = P_2A - P_1A = \Delta P\cdot A$

svo $F_G = 0{,}6\cdot 101{,}3\cdot 10^3\cdot 0{,}023 = 1400$ N

10.D:

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$ þ.a. $v = \sqrt{\frac{k}{m}}\cdot x \approx 0{,}47$ m/s.

11. C:



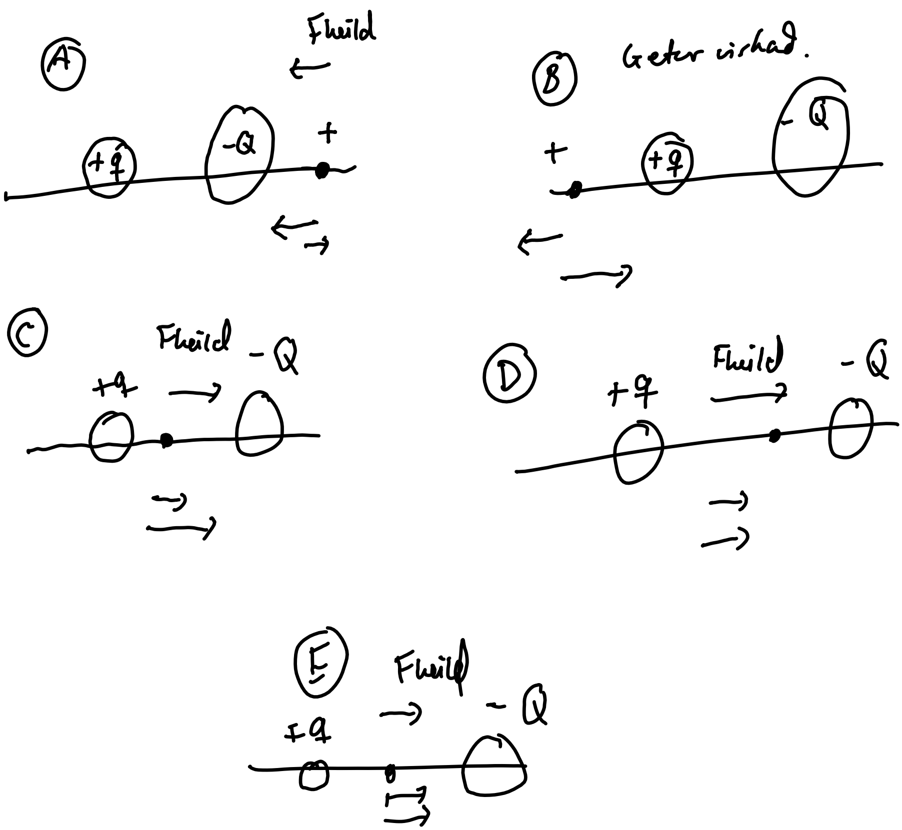
$$T + F_{upp} = mg$$

p.a. $T = mg - F_{upp} = mg - \rho_{vatn} \cdot V_{stytta}\, g$

ew $\rho := \frac{m}{V}$ p.a. $V_{stytta} = \frac{m}{\rho_{stytta}}$ p.a.

$$T = \left(1 - \frac{\rho_{vatn}}{\rho_{gull}}\right) m_{stytta} \cdot g = 230\ N.$$

12. B



13. A

Höfum þá að

$$mgh = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

og $v_{cm} = r\omega$ því þeir rúlla án þess að renna svo

$$mgh = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \frac{v_{cm}^2}{r^2}$$

skrifum $I = \gamma m r^2$ þá fæst:

$$mgh = \frac{1}{2}(1+\gamma) m v_{cm}^2 \quad \text{svo} \quad \boxed{v_{cm} = \sqrt{\frac{2mgh}{1+\gamma}}.} \quad (*)$$

Fyrir stálkúlu er $\gamma = \frac{2}{5}$

Fyrir giftingarhring er $\gamma = 1$

Fyrir kefli er $\gamma = \frac{1}{2}$

Ljóst er af (*) að v_{cm} eykst ef γ minnkar.

en $\frac{2}{5} < \frac{1}{2} < 1$ svo röðin er

stálkúlan fyrst, svo keflið, loks hringurinn.

<u>14. A:</u>

$$P = IV = 19 \cdot 5 = 95\,\mathrm{W}.$$

<u>15. E:</u>

$$a = \frac{r_{min} + r_{max}}{2} \quad \text{og} \quad \frac{a^3}{T^2} = \frac{a_{Jörð}^{\;3}}{T_{jörð}^{\;2}}$$

$$\text{svo} \quad T = \left(\frac{a}{a_{Jörð}}\right)^{3/2} \text{ár} = \left(\frac{r_{min} + r_{max}}{2 \cdot a_{jörð}}\right)^{3/2} \text{ár} = 248 \text{ ár}.$$

<u>16. B</u>

$$\Delta\theta = \frac{1}{2}\alpha t^2 \quad \text{svo} \quad t = \sqrt{\frac{2\Delta\theta}{\alpha}} = 20{,}2 \text{ s}$$

þar til það líður yfir kanínuna.

en $\quad w = \alpha t \quad$ svo $\quad t = \frac{w}{\alpha} = \frac{35}{2} = 17{,}5$ s

þar til líður yfir hvolpinn. Þá hefur hringekjan farið $\frac{\Delta\theta}{2\pi} = \frac{\frac{1}{2}\alpha t^2}{2\pi} = \frac{w^2}{4\pi\alpha} = 48$ hringi.

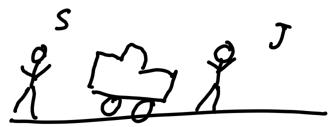
17. B

$$\frac{L_B}{L_A} = \frac{m v_B r_B}{m v_A r_A} \quad \text{og} \quad r_B = 2r_A \quad \text{og} \quad m\frac{v^2}{r} = \frac{GMm}{r^2}$$

svo $v_A = \sqrt{\frac{2GM}{r_A}}$ og $v_B = \sqrt{\frac{2GM}{r_B}}$ þ.a.

$$\frac{L_A}{L_B} = \sqrt{\frac{r_B}{r_A}} = \sqrt{\frac{2r_A}{r_A}} = \sqrt{2}.$$

18. D.



19. A:

$$m_F v_F = (m_F + m_f)v \quad \text{svo} \quad v = \frac{m_F v_F}{(m_f + m_F)}$$

$$\Delta K = \frac{1}{2} m_F v_F^2 - \frac{1}{2}(m_F + m_f) v^2 = \frac{1}{2} m_F v_F^2 \left(1 - \frac{m_F}{m_f + m_F}\right) = 266 \text{ kJ}$$

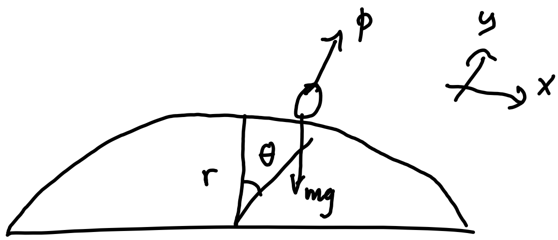
20. A:

$F_{mið} = m\frac{v^2}{r}$ og $2g\Delta s = v^2$ þ.a. $\Delta s = \frac{v^2}{2g} = \frac{F_{mið}\cdot r}{2mg}$

en heildarhæðin er þá $h = r + \frac{F_{mið}\cdot r}{2mg} = 26$ m.

Seinni hluti:

Ögn sem rennur af kúlu



(a) $mgr = mgr\cos\theta + \frac{1}{2}mv^2$

þ.a. $v = \sqrt{2gr(1-\cos\theta)}$.

(b) Þegar hún losnar er $p=0$. Þangað til er:

$$\begin{pmatrix} ma_x \\ m\frac{v^2}{r} \end{pmatrix} = \begin{pmatrix} mg\sin\theta \\ mg\cos\theta - p \end{pmatrix} \quad \text{svo} \quad p = mg\cos\theta - m\frac{v^2}{r}$$

svo $p = mg\cos\theta - 2mg(1-\cos\theta) = 3mg\cos\theta - 2mg$.

(c) $p=0$ svo $\cos\theta = \frac{2}{3}$ þ.a. $\theta = \arccos\left(\frac{2}{3}\right) = 48{,}2^\circ$.

Hönnun valslöngva:

(a) $I = m l_1^2 + M l_2^2.$

(b) Honum er þá sleppt svona: 

Fáum:

$$0 = mgl_1 - Mgl_2 + \frac{1}{2} I \omega^2$$

svo $$\omega = \sqrt{\frac{2g(Ml_2 - ml_1)}{I}} = \sqrt{\frac{2g(Ml_2 - ml_1)}{ml_1^2 + Ml_2^2}}.$$

(c) Þá er $v = l_1 \omega$ og

$$\begin{pmatrix} s \\ l_1 + h \end{pmatrix} = \begin{pmatrix} vt \\ \frac{1}{2} g t^2 \end{pmatrix}$$ svo $$t = \sqrt{\frac{2(l_1+h)}{g}}$$ og þá

$$s = vt = l_1 \omega t = l_1 \cdot \sqrt{\frac{2g(Ml_2 - ml_1)}{ml_1^2 + Ml_2^2}} \cdot \sqrt{\frac{2(l_1+h)}{g}}$$

$$= l_1 \cdot \sqrt{\frac{4(Ml_2 - ml_1)(l_1+h)}{ml_1^2 + Ml_2^2}} \quad (d) \quad = 49{,}8 \text{ m}.$$